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RESONANCE FLUORESCENCE OF A TRAPPED FOUR–LEVEL ATOM WITH BICHROMATIC DRIVING*

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The RF spectrum of a bichromatically driven four–level atom is polarization dependent. Very narrow lines occur in the incoherent parts of the spectrum for polarization directions which are different from that of the driving fields. The degree of squeezing has a maximum of 56% which should make it easily observable. The second–order correlation function exhibits antibunching for zero time delay and strong superbunching for certain values of the interaction parameter and time delay. For these parameters resonant two–photon emission takes place in the form of polarization entangled photon pairs. The system can be a novel source of photons in the EPR and/or Bell states. Some experiments will be proposed which make use of this unique source.

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1 Introduction

Strongly correlated photons are the basic requirement for the generation of squeezing. For the production of EPR–pairs one needs, in addition, the presence of different polarization directions in the photon field in most cases. Both conditions can be met in the RF of a bichromatically driven four–level atom. The four–level atom has two twofold degenerate states where the degeneracy is with respect to the magnetic quantum number $m_j = \pm 1/2$ [1]. As a consequence of the internal structure of the four–level atom, different polarization directions are now possible in the RF and photons with different polarization directions are entangled.

A recent paper by Eichmann *et al.* [2] reported on interference experiments using the light scattered from two single $^{189}Hg^{+}$ ions in a linear trap. These ions have an internal structure identical to the 4–level atom we are considering in this paper. We showed that the system is

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appropriate for the realization of the quantum eraser [3]. A similar experimental setup would make the verification of squeezing [4] and entanglement [5] feasible. Many applications require appropriate superpositions of EPR– and/or Bell–states which, in the case of two trapped Hg ions, can be ralized if we drive one of them with an x–polarized field and the other with a z–polarized field. Another important feature of this system is that we are in the steady–state regime of the atoms and, therefore, the generated two–photon states are free of decoherence as long as there are no other absorption or scattering processes.

The paper is organized as follows. In Sec. 2 we present the Floquet-states for the interaction of the four-level atom with the z-polarized bichromatic driving field. The symmetric level configuration of the four-level atom, as a consequence of the degeneracy of the atomic levels, leads to similar Floquet-states to those of a bichromatically driven two-level atom. Then the master equations in Floquet-state representation are obtained, taking into account the secular approximation. In Sec. 3 we calculate the polarization dependent RF spectra and squeezing properties. In Sec. 4 we study pair correlations in the RF field with emphasis on the different polarization directions. Sec. 5 summarizes our conclusions.

2 Floquet-states of the four-level atom and the master equation

We consider a four-level atom with two twofold degenerated upper and lower levels with respect to the magnetic quantum number $m_j = \pm 1/2$, as indicated in Fig. 1. Consequently, the spontaneous emission event consists of π- ($\Delta m_j = 0$) and σ- ($\Delta m_j = \pm 1$) transitions. The dipole moment operators were obtained in Ref. [1]. We assumed the same magnitude for the different dipole moments, $|d_x| = |d_y| = |d_z| \equiv d$, to obtain

$$d_x^\downarrow = (|1\rangle\langle 4| + |2\rangle\langle 3|)d, \tag{1a}$$
$$d_y^\downarrow = -i\,(|1\rangle\langle 4| - |2\rangle\langle 3|)\,d, \tag{1b}$$
$$d_z^\downarrow = (|2\rangle\langle 4| - |1\rangle\langle 3|)d, \tag{1c}$$
$$d_{x,y,z}^\uparrow = (d_{x,y,z}^\downarrow)^\dagger. \tag{1d}$$

The four-level atom is driven with a z-polarized (π-transitions) bichromatic field with equal amplitudes symmetrically detuned around the atomic transition ω_0 (Fig. 2),

$$\vec{E}(t) = E_0 \mathrm{Re}(e^{i\omega_{1L}t} + e^{i\omega_{2L}t - 2i\phi})\vec{e}_z, \tag{2}$$

where, the frequencies are given as $\omega_{1L} = \omega_0 - \delta$ and $\omega_{2L} = \omega_0 + \delta$ and ϕ is a relative phase between the two field components. The interaction between the four-level atom and the bichromatic field can be solved analytically with a Floquet-ansatz, since the atomic state $|1\rangle$ is only coupled to the atomic state $|3\rangle$ and the atomic state $|2\rangle$ is only coupled to the atomic level $|4\rangle$. There is no coupling between these two subsystems and, consequently, they can be treated separately. This leads to the same equations as the ones for a bichromatically driven two-level atom. For details see Ref. [6]. The Floquet-states can be described in rotating wave approximation by

$$|\Phi_1(t)\rangle = A_1(t)\,|1\rangle + iA_2(t)e^{-i(\omega_0 t+\phi)}\,|3\rangle \tag{3a}$$
$$|\Phi_2(t)\rangle = A_1(t)\,|2\rangle - iA_2(t)e^{-i(\omega_0 t+\phi)}\,|4\rangle \tag{3b}$$

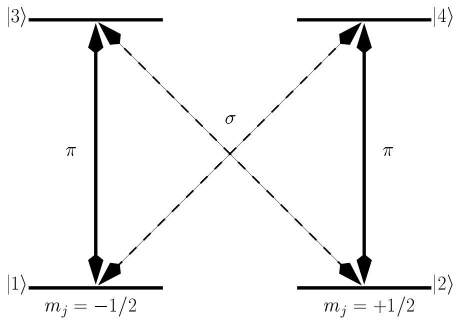

Fig. 1. Level structure of the four-level atom with the different polarization dependent decay channels. The σ channels are connected with x- and y-polarized light while the π transition is connected with z-polarized light.

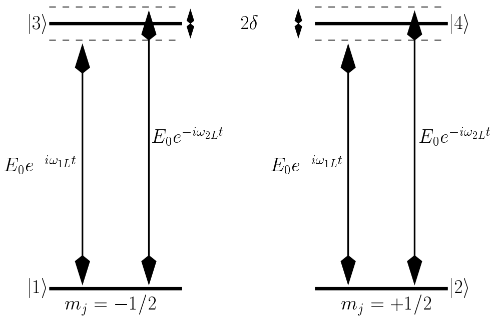

Fig. 2. Interaction of the z-polarized bichromatic field and the four-level atom. The bichromatic field consists of two components of equal amplitudes symmetrically detuned around the atomic transition frequency.

$$|\Phi_3(t)\rangle = +iA_2(t)e^{i\phi}\,|1\rangle + A_1(t)e^{-i\omega_0 t}\,|3\rangle \tag{3c}$$
$$|\Phi_4(t)\rangle = -iA_2(t)e^{i\phi}\,|2\rangle + A_1(t)e^{-i\omega_0 t}\,|4\rangle \tag{3d}$$
$$A_1(t) = \cos\left[(\xi/2)\sin\left(\delta t + \phi\right)\right] \tag{3e}$$
$$A_2(t) = \sin\left[(\xi/2)\sin\left(\delta t + \phi\right)\right], \tag{3f}$$

where the interaction parameter ξ is defined as $\xi = 2V_0/(\hbar\delta)$, and $V_0 = |\vec{d}\cdot\vec{E}_0|$. The difference between the signs in $|\Phi_1(t)\rangle$ and $|\Phi_2(t)\rangle$ and $|\Phi_3(t)\rangle$ and $|\Phi_4(t)\rangle$ is a consequence of the dipole moment operator (1c) which has different signs for the atomic transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ for the dipole moment in z-direction.

Using Eqs. (3a)-(3f), we can express the dipole moment operators in Floquet-state representation. In general the matrix elements of the dipole transitions are given as

$$\vec{d}_{\alpha\beta}(t) = \langle\Phi_\alpha(t)|\,\vec{d}\,|\Phi_\beta(t)\rangle = \vec{d}^{(-)}_{\alpha\beta}(t) + \vec{d}^{(+)}_{\alpha\beta}(t), \tag{4}$$

where $\vec{d}^{(\pm)}_{\alpha\beta}(t)$ are the positive- and negative-frequency parts of the dipole matrix element, leading to

$$
\begin{aligned}
d^{x(-)}_{12}(t) &= -iA_1(t)A_2(t)e^{-i\omega_0 t-i\phi}d = d^{x(-)}_{34}(t), && \text{(5a)}\\
d^{x(-)}_{21}(t) &= +iA_1(t)A_2(t)e^{-i\omega_0 t-i\phi}d = d^{x(-)}_{43}(t), && \text{(5b)}\\
d^{x(-)}_{23}(t) &= +[A_1(t)]^2e^{-i\omega_0 t}d = d^{x(-)}_{14}(t), && \text{(5c)}\\
d^{x(-)}_{32}(t) &= -[A_2(t)]^2e^{-i\omega_0 t-2i\phi}d = d^{x(-)}_{41}(t), && \text{(5d)}
\end{aligned}
$$

for x-polarization (and, except for some unimportant phase factors, an identical set for the y-polarization), and

$$
\begin{aligned}
d^{z(-)}_{11}(t) &= -iA_1(t)A_2(t)e^{-i\omega_0 t-i\phi}d = d^{z(-)}_{22}(t), && \text{(6a)}\\
d^{z(-)}_{33}(t) &= +iA_1(t)A_2(t)e^{-i\omega_0 t-i\phi}d = d^{z(-)}_{44}(t), && \text{(6b)}\\
d^{z(-)}_{13}(t) &= -[A_1(t)]^2e^{-i\omega_0 t}d = -d^{z(-)}_{24}(t), && \text{(6c)}\\
d^{z(-)}_{31}(t) &= -[A_2(t)]^2e^{-i\omega_0 t-2i\phi}d = -d^{z(-)}_{42}(t), && \text{(6d)}
\end{aligned}
$$

for z-polarization. For the positive frequency parts of the dipole matrix elements $d^{(+)}_{\alpha\beta}(t) = [d^{(-)}_{\beta\alpha}(t)]^*$ holds.

Transition between the Floquet-states (3a)-(3f) can be induced by the interaction of the combined "atom plus bichromatic field" system with a reservoir. The master equations in the Floquet-representation in general form were investigated in Ref. [7] in the secular or separated spectral lines approximation which holds if the detuning δ of the driving field components is much bigger than the decay rate γ of the atomic transitions. We get for our case, under these conditions, the following set of master equations for the reduced density operator elements,

$$
\begin{aligned}
\frac{\partial}{\partial t}\sigma_{11,22}(t) &= -\frac{\gamma}{6}\big\{a_1(\xi)\sigma_{11,22}(t) - \frac{1}{2}b(\xi)\sigma_{22,11}(t)\\
&\qquad -\frac{1}{2}c_1(\xi)[\sigma_{44,33}(t) + \frac{1}{2}\sigma_{33,44}(t)]\big\}, && \text{(7a)}\\
\frac{\partial}{\partial t}\sigma_{33,44}(t) &= -\frac{\gamma}{6}\big\{a_2(\xi)\sigma_{33,44}(t) - \frac{1}{2}b(\xi)\sigma_{44,33}(t)\\
&\qquad -\frac{1}{2}c_2(\xi)[\sigma_{22,11}(t) + \frac{1}{2}\sigma_{11,22}(t)]\big\}, && \text{(7b)}
\end{aligned}
$$

for the populations, and,

$$
\begin{aligned}
\frac{\partial}{\partial t}\sigma_{12}(t) &= -\frac{\gamma}{6}\big\{a_1(\xi)\sigma_{12}(t) + \frac{1}{4}c_1(\xi)\sigma_{34}(t)\big\}, && \text{(8a)}\\
\frac{\partial}{\partial t}\sigma_{34}(t) &= -\frac{\gamma}{6}\big\{a_2(\xi)\sigma_{34}(t) + \frac{1}{4}c_2(\xi)\sigma_{12}(t)\big\}, && \text{(8b)}\\
\frac{\partial}{\partial t}\sigma_{13}(t) &= -\frac{\gamma}{6}\big\{d(\xi)\sigma_{13}(t) - \frac{1}{2}b(\xi)\sigma_{24}(t)\big\}, && \text{(8c)}\\
\frac{\partial}{\partial t}\sigma_{24}(t) &= -\frac{\gamma}{6}\big\{d(\xi)\sigma_{24}(t) - \frac{1}{2}b(\xi)\sigma_{13}(t)\big\}, && \text{(8d)}
\end{aligned}
$$

$$\frac{\partial}{\partial t}\sigma_{23}(t) = -\frac{\gamma}{6}\{d(\xi)\sigma_{23}(t)\}, \tag{8e}$$
$$\frac{\partial}{\partial t}\sigma_{14}(t) = -\frac{\gamma}{6}\{d(\xi)\sigma_{14}(t)\}, \tag{8f}$$
$$\frac{\partial}{\partial t}\sigma^*_{\alpha\beta}(t) = \frac{\partial}{\partial t}\sigma_{\beta\alpha}(t) \qquad \text{where } \alpha \neq \beta, \tag{8g}$$

for the correlations. The coefficients, which depend on the atom-field interaction strength ξ, are of the following form,

$$a_{1,2}(\xi) = [11/4 \mp 3J_0(\xi) + (1/4)J_0(2\xi)], \tag{9a}$$
$$b(\xi) = [1 - J_0(2\xi)], \tag{9b}$$
$$c_{1,2}(\xi) = [3 \pm 4J_0(\xi) + J_0(2\xi)], \tag{9c}$$
$$d(\xi) = [13/4 - (1/4)J_0(2\xi)]. \tag{9d}$$

Here, J_0 is the Bessel function of zeroth order.

3 Spectral and squeezing properties

The polarization dependent RF spectra can be obtained by taking the Fourier transform of the first order autocorrelation function. As usual, we express the fields in the correlation function by their sources, the dipole moment operators. With the help of Eqs. (5a)-(6d), for the dipole moments in Floquet-representation, and Eqs. (7a)-(7b) and (8a)-(8g), for the time evolution of the populations and correlations of the density operator, a lengthy but straightforward calculation yields

$$\begin{aligned} S_z(\omega_k) &= S_{z1}(\omega_k) + S_{z2}(\omega_k), \\ S_{z1}(\omega_k) &\propto \sum_q J^2_{2q+1}(\xi)\Delta^2\delta(\omega_k - \omega_{2q+1}), \\ S_{z2}(\omega_k) &\propto \sum_q \Bigg\{[\delta_{q0}(1 - 2J_0(\xi)\Delta) + J^2_{2q}(\xi)]\, \frac{\Gamma^{(z)}_{12}}{(\omega_k - \omega_{2q})^2 + (\Gamma^{(z)}_{12})^2} \\ &\quad + J^2_{2q+1}(\xi)\frac{(1-\Delta^2)\Gamma^{(z)}_{11}}{(\omega_k - \omega_{2q+1})^2 + (\Gamma^{(z)}_{11})^2}\Bigg\}, \end{aligned} \tag{10}$$

for the z-polarized part of the spectrum where $S_{z1}(\omega_k)$ stands for the elastic part and$S_{z2}(\omega_k)$ for the inelastic part. For the x- and y-polarized parts of the spectra, which consist only of incoherent terms, we obtain

$$\begin{aligned} S_{x,y}(\omega_k) \propto \sum_q \Bigg\{ &[\delta_{q0}(1 - 2J_0(\xi)\Delta) + J^2_{2q}(\xi)]\, \frac{\Gamma^{(x,y)}_{12}}{(\omega_k - \omega_{2q})^2 + (\Gamma^{(x,y)}_{12})^2} \\ + &\frac{J^2_{2q+1}}{2}(\xi)\Bigg[\big(1 + \frac{1}{\sqrt{1+8\Delta^2}} - \frac{4\Delta^2}{\sqrt{1+8\Delta^2}}\big)\frac{\Gamma^{(1)(x,y)}_{11}}{(\omega_k - \omega_{2q+1})^2 + (\Gamma^{(1)(x,y)}_{11})^2} \\ &+\big(1 - \frac{1}{\sqrt{1+8\Delta^2}} + \frac{4\Delta^2}{\sqrt{1+8\Delta^2}}\big)\frac{\Gamma^{(2)(x,y)}_{11}}{(\omega_k - \omega_{2q+1})^2 + (\Gamma^{(2)(x,y)}_{11})^2}\Bigg]\Bigg\}. \end{aligned} \tag{11}$$

Here again $J_q(\xi)$ is the Bessel function of order q. We have introduced in the above equations the steady state population difference $\Delta = \sigma_{11}(\infty) + \sigma_{22}(\infty) - [\sigma_{33}(\infty) + \sigma_{44}(\infty)]$ as well as different decay rates, given by

$$\begin{aligned}
\Delta &= \frac{4J_0(\xi)}{3 + J_0(2\xi)}, \\
\Gamma_{11}^{(z)} &= \frac{\gamma}{4}[3 + J_0(2\xi)], \\
\Gamma_{12}^{(z)} &= \frac{\gamma}{8}[5 - J_0(2\xi)], \\
\Gamma_{11}^{(1)(x,y)} &= \frac{\gamma}{6}\left\{[\frac{11}{4} + \frac{1}{4}J_0(2\xi)] + \sqrt{1 + 8\Delta^2}\frac{J_0(\xi)}{\Delta}\right\}, \\
\Gamma_{11}^{(2)(x,y)} &= \frac{\gamma}{6}\left\{[\frac{11}{4} + \frac{1}{4}J_0(2\xi)] - \sqrt{1 + 8\Delta^2}\frac{J_0(\xi)}{\Delta}\right\}, \\
\Gamma_{12}^{(x,y)} &= \frac{\gamma}{6}[\frac{13}{4} - \frac{1}{4}J_0(2\xi)].
\end{aligned} \tag{12}$$

The frequencies ω_k, ω_{2q}, and ω_{2q+1} obey the relations

$$\omega_k = \omega - \omega_0, \tag{13a}$$
$$\omega_{2q} = \omega_0 - 2q\delta, \tag{13b}$$
$$\omega_{2q+1} = \omega_0 - (2q + 1)\delta, \tag{13c}$$

so that the spectra are displayed around the atomic transition frequency ω_0.

Here we give only a qualitative description of the power spectrum. For a more quantitive treatment see Ref. [4]. The spectrum in the polarization direction of the z-polarized driving field contains a coherent part and is exactly the same as the spectrum for a bichromatically driven two-level atom, due to the degeneracy of the four-level atom. However, the σ-polarized part of the spectrum displays some novel features when compared to the z-polarized part. First, the coherent part is missing. For small interaction parameters, $\xi \ll 1$, the sidebands of the spectra at frequencies ω_{2q+1} become very narrow. This behavior is a consequence of the fact that the spectral linewidths for the sidebands contain the absorption linewidth for small ξ which is of the order of $(\xi/\delta)^2\gamma$. On the other hand, the absorption linewidth plays no role in the central peak component of the spectrum. Furthermore, the light emitted into the sidebands of the spectra is classical for small interaction parameters. In the range of larger interaction parameters the differences between the z- and x- or y-polarized parts of the spectrum tend to diminish.

A note on our choice of the coordinate system: The z-axis is defined by the polarization direction of the driving field. The x-axis is defined as the direction of propagation of the driving field. The RF consists of three different, orthogonal polarizations, the z- (or π-) polarized component which contains the coherent part and the incoherent x- and y- (or σ-) polarized components. The results for the spectral and squeezing properties of the x- and y-polarized components of the RF are identical.

Next, we consider the squeezing properties of the bichromatically driven four-level atom. Usually squeezing is detected with a homodyne detection scheme [8] where the signal field is combined with a strong local oscillator (LO, a classical field) by a beam splitter and we assume that the phase Θ of the LO can be varied arbitrarily relative to the signal beam. To simplify the calculations we assume that we filter out certain linear polarization directions of the signal, i.e. the RF, with the help of a linear polarizer and use a strong local field of the same linear polarization, in addition.

Squeezing in the signal beam is conveniently expressed in terms of the normalized intensity correlation function $\lambda(\tau, \Theta)$ [9, 10]

$$\begin{aligned}\lambda(\tau,\Theta) &= \langle \Delta E^{(-)}(t)\Delta E^{(+)}(t+\tau)\rangle e^{(i\omega_0\tau)} \\ &+ \langle \Delta E^{(-)}(t)\Delta E^{(-)}(t+\tau)\rangle e^{(-i[\omega_0(2t+\tau)-2\Theta])} + \text{ c.c.,}\end{aligned} \tag{14}$$

where $\Delta E = E - \langle E\rangle$ and $E^{(\pm)}(t)$ is the said x-, y- or z-polarized plane wave expansion of the signal field. Squeezing occurs when $\lambda(\tau = 0, \Theta) < 0$ for some values of the angle Θ [11]. The time evolution of $\lambda(\tau, \Theta)$ describes the progressive loss of negative correlation with increasing delay-time. The Fourier transformation of $\lambda(\tau, \Theta)$ is connected with spectral component squeezing as a function of Θ,

$$\Psi(\omega,\Theta) = \int_{-\infty}^{\infty} \lambda(\tau,\Theta)e^{i\omega\tau}d\tau. \tag{15}$$

Spectral component squeezing is obtained if $\Psi(\omega, \Theta) < 0$ for some values of ω and the phase Θ of the local oscillator.

Squeezing occurs if $\mathrm{Q}_{x,y,z}(\Theta) = \lambda_{x,y,z}(\tau = 0, \Theta) < 0$. We assume that the relative phase between the two driving field components is equal to zero. Usually, one considers the two quadrature components of the signal field which are in phase ($\Theta = 0$) and out-of-phase ($\Theta = \pi/2$) with the strong local oscillator, respectively. If we assume an ideal detector with 100% quantum efficiency that collects all of the RF over the total surface angle of 4π, we get for $Q_{x,y,z}(\Theta = \pi/2)$ the expressions,

$$Q_z(\Theta = \frac{\pi}{2}) = \frac{\gamma}{2}\{2 - 2\Delta J_0(\xi) - \Delta^2[1 - J_0(2\xi)]\}, \tag{16}$$

for the z-polarized part, and

$$Q_{x,y}(\Theta = \frac{\pi}{2}) = \frac{\gamma}{2}\{1 + J_0(2\xi) - 2\Delta J_0(\xi)\}, \tag{17}$$

for the x- and y-polarized parts of the RF. The correlation functions $Q_{x,y,z}(\Theta)$ are normalized with respect to the total outgoing flux γ. Similarly, we obtain the following expression for the in-phase quadrature of the signal, again assuming an ideal detector,

$$Q_z(\Theta = 0) = \gamma[1 - \Delta J_0(\xi)], \tag{18}$$

for the z-polarized part, and

$$Q_{x,y}(\Theta = 0) = \frac{\gamma}{2}[2 - 2\Delta J_0(\xi) + 1 - \Delta J_0(2\xi)], \tag{19}$$

for the x-and y-polarized parts of the RF. The in-phase quadrature of the signal can not be squeezed regardless of the polarization direction. However, the out-of-phase components (16), (17) display squeezing. It is interesting to note that the z-polarized part of the RF of the four-level atom displays the same squeezing properties as those of a bichromatically driven two-level atom [6]. However, the x- and y- polarized parts of the RF or the part of the RF with polarization directions different from that of the driving field, in general, show different squeezing properties. The x- and y-polarized parts are stronger squeezed and, in addition, display squeezing over a larger range of the interaction parameter.

We mention also that the squeezed quadrature component depends on the relative phase ϕ of the driving fields. When $\phi \neq 0$ the quadrature component $\pi/2 - \phi$ is squeezed. Therefore we can determine the squeezed quadrature component with the help of the relative phase of the driving fields.

As mentioned already, the spectrum of squeezing or the degree of squeezing is connected to the Fourier transform of the correlation function. The polarization dependent squeezing spectrum for the out-of-phase component ($\Theta = \pi/2$), which we denote as $\Phi_{22,x,y,z}(\omega)$, is given in the case of the relative phase $\phi = 0$ of the driving fields by

$$\begin{aligned} \Phi_{22,z}(\omega) &= \sum_{q=-\infty}^{+\infty} [J_{2q}^2(\xi) - \delta_{q,0}\Delta J_0(\xi)] \frac{2\Gamma_{12}^{(z)}\gamma}{(\omega - 2q\delta)^2 + (\Gamma_{12}^{(z)})^2} \\ &+ \sum_{q=-\infty}^{+\infty} J_{2q+1}^2(\xi)(1-\Delta^2) \frac{2\Gamma_{11}^{(z)}\gamma}{[\omega - (2q+1)\delta]^2 + (\Gamma_{11}^{(z)})^2}, \end{aligned} \tag{20}$$

for the z-polarized part, and

$$\Phi_{22,x,y}(\omega) = \sum_{q=-\infty}^{+\infty} [J_{2q}^2(\xi) - \delta_{q,0}\Delta J_0(\xi)] \frac{2\Gamma_{12}^{(x,y)}\gamma}{(\omega - 2q\delta)^2 + (\Gamma_{12}^{(x,y)})^2}, \tag{21}$$

for the x- and y-polarized parts of the RF. Spectral component squeezing occurs only in the central peak component of the RF spectra, independently of the polarization directions.

The x- and y-polarized part of the RF shows a stronger degree of squeezing of the central peak component of the spectrum than the z-polarized part. This is the result of the different decay rates $\Gamma_{12}^{(x,y)}$ and $\Gamma_{12}^{(z)}$. We can approximate the degree of squeezing of the central peak component due to the separated lines approximation as

$$\Phi_{22,z}(0) \approx 2[J_0^2(\xi) - \Delta J_0(\xi)](\gamma/\Gamma_{12}^{(z)}). \tag{22}$$

for the z-polarized part and

$$\Phi_{22,x,y}(0) \approx 2[J_0^2(\xi) - \Delta J_0(\xi)](\gamma/\Gamma_{12}^{(x,y)}). \tag{23}$$

for the x- and y-polarized part of the spectrum. The ratio of the degree of squeezing between z- and x- or y-polarized RF light is given as $\Gamma_{12}^{(z)}/\Gamma_{12}^{(x,y)}$. This ratio is approximately 1/1.15 for the interaction strength ξ where the maximum degree of squeezing occurs. When we take into account the already very high degree of squeezing of the z-polarized part (which is the same as that of a bichromatically driven two-level atom) of approximately 48% we get a maximum degree of squeezing of around 56% in the x- and y-polarized part of the RF.

In Figs. 3, we have plotted the degree of squeezing for the different polarization directions as function of ω and the interaction strength ξ. Squeezing only occurs in the central peak component of the spectrum. No squeezing occurs in the in phase component regardless of

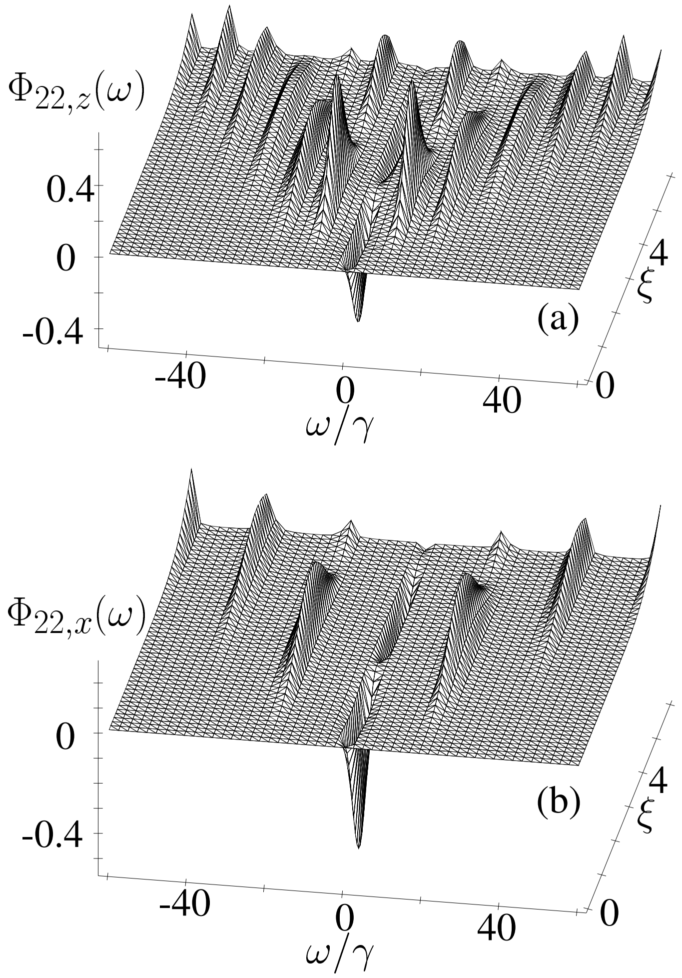

Fig. 3. Spectral component squeezing vs. ξ and ω for the out-of-phase components ($\Theta = \pi/2$) for (a) the z-polarized, and (b) the x- or y-polarized part of the RF light, with detuning $\delta/\gamma = 10$.

the polarization directions and frequencies. We stress once again that the squeezed quadrature component can be determined with the relative phase ϕ of the driving fields.

The main advantage of the bichromatically driven four-level atom for the study of the squeezing properties as compared to the bichromatically driven two-level atom is the fact that strongest squeezing occurs in the polarization component which is perpendicular to that of the driving field. This allows us, in an unambiguous manner, to separate the contribution to the RF which displays strongest squeezing effects. In this context, it should be noted that the first observation of squeezing, in the RF of monochromatically driven two-level atoms, has been reported very recently [12].

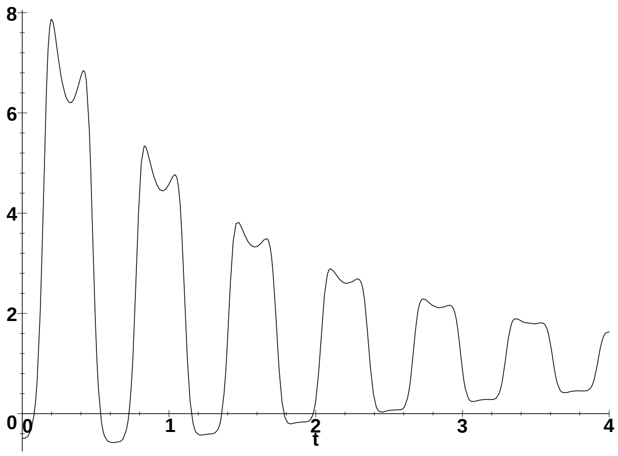

Fig. 4. Second order correlation function of σ^+ and σ^- photons in the case of z–polarized driving fields for interaction strength $\xi = 0.1$.

4 Pair production

In this section we demonstrate the generation of strongly correlated photon pairs in the RF of a bichromatically driven four–level atom and show that photons of different polarization directions are entangled. We are interested in the polarization dependent photon correlations since we want to realize EPR–pairs or, more generally, to generate Bell–states in the resonance fluorescence of the bichromatically driven four–level atom. We expect that, for certain values of the interaction parameter, a favorable situation arises and the relative weight of the two–photon emission processes, which are already present in the bichromatically driven two–level atom, can be enhanced.

The normalized second–order correlation function can be expressed analytically by making use of the quantum–regression theorem and the secular approximation. The analytical expressions are, however, much too complicated in order to get any useful insight from them. Therefore we confine the present treatment to numerical results only.

We consider, for example, the correlations in two–photon emissions for a pair of σ^+ and σ^- circularly polarized photons in the case of a z–polarized bichromatic driving field. Fig. 4 shows very strong super–bunching for small ξ. Such a strong super–bunching behavior is the result of a two–photon resonant emission process of a σ^+–σ^- or σ^-–σ^+ circularly polarized photon pair. This two–photon emission process strongly depends on the steady–state populations of the atomic ground–states $|1\rangle$ and $|2\rangle$ and is most probable for those values of ξ for which the atomic ground level population and, hence, the steady–state population difference is a maximum. In contrast, we expect no strong super–bunching behavior for those values of ξ where $\Delta = 0$ or, in other words, when suppression of the two–photon emission processes occurs. In comparison, there are no super–bunching effects for a σ^+–σ^+ or a σ^-–σ^- photon emission process for the entire range of the interaction parameter.

We consider, next, the arising field states more precisely and discuss the case of a z–

polarized driving field. In the previously discussed configuration, we observe the correlated photons in opposite z–directions to get rid of possible z–polarized photons. In general, there are two possible polarizations for each photon individually. The first possibility is that each photon by itself is linearly polarized, the second possibility is that each photon by itself is unpolarized. In the first case, the probability for an EPR–photon pair in the long–time regime is $1/2$.

In the case when the individual photons are unpolarized, only the two–photon emission terms of the σ^+ and σ^- polarized photons survive and contribute to the second–order correlation function. We must average over all possible arbitrary phase factors of the one–photon transitions. As a consequence the arising photon state is given as

$$\Big(|\Psi\rangle_{\mathrm{unpol}}\Big)_{\mathrm{z-driv}} = \frac{1}{\sqrt{2}}\left[|1_{+,+z}, 0_{-,+z}, 0_{+,-z}, 1_{-,z}\rangle + |0_{+,+z}, 1_{-,+z}, 1_{+,-z}, 0_{-,z}\rangle\right]. \tag{24}$$

In other words, we have a perfect EPR–correlated photon pair of circularly polarized photons. It is easy to show that each photon is unpolarized by calculating the polarization matrix for one photon and tracing over the other [10].

The experimental setup of Ref. [2] would make the verification of the above results feasible and would even offer further opportunities for quantum computing and quantum information processing since these applications require appropriate superpositions of the above EPR– and Bell–states. Perhaps the most important feature of this system is that we are in the steady–state regime of the atoms and, therefore, the generated two–photon states are completely free of decoherence as long as there are no other absorption or scattering processes.

5 Conclusions

We have considered the resonance fluorescence spectra and squeezing properties of a bichromatically driven four-level atom. Most interesting is the remarkably high degree of squeezing of approximately 56% in the polarization component of the RF which is different from that of the driving field and the generation of entangled photon pairs.

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