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QUANTUM STATE ENGINEERING WITH PHOTONIC QUBITS*

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We outline a scheme for the generation of a train of entangled single-photon wavepackets using standard CQED-techniques. The generated photons are transferred to the continuum outside the resonator through cavity loss in the form of wavepackets each of which may be regarded as a logical qubit. We show that undesired decoherence effects can be efficiently reduced in the considered scheme.

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1 Introduction

A controlled source of entangled qubits is necessary for most quantum communication and computation procedures [1, 2]. For communication purposes the choice of photons as qubits is especially appropriate, since they propagate reliably over long distances [3]. In this sense, an important experimental step has been taken in quantum communication with the recent successful teleportation of a qubit between two spatially distant locations [4, 5]. The quintessential ingredient to a successful implementation of this quantum communication process was the availability of a source of entangled qubits. The source presently in use is parametric downconversion in a crystal, which is a reliable but random and largely untailorable source of entangled twin-photons. Although there is an intrinsic limitation on the number of entangled qubits that can be produced directly by downconversion, novel schemes have been found to extract a GHZ state from two pairs of twin-photons by measuring the state of one qubit [6]. Yet the efficiency of this process is low and it is foreseeable that for instance teleporting simultaneously several qubits will require a novel source of entangled qubits.

Here we discuss a scheme for the generation of a train of entangled single-photon wavepackets which are well resolved in time [7] this allowing us us to regard them as individual qubits. Our scheme is based on well-established techniques from cavity quantum electrodynamics (CQED) [8–10]. We consider a nonclassical medium (for instance, an atom or ion) inside a high quality optical resonator. With the use of external pump fields, the coupling of the active medium to the resonator allows a single photon transfer to the resonator and therefrom via cavity decay to the continuum outside. The process may be regarded as a *quantum data bus*

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where the logical state is encoded in the active medium and then mapped onto a photonic qubit generated in the continuum outside the resonator. Before the process is repeated the medium is coherently recycled to its initial state, which provides the basis for a build-up of entanglement between subsequently generated photonic qubits. An encoding of quantum information in the one-photon wavepackets could either take place by identifying two orthogonal polarization states of the single photon with logical "0" and "1", or by regarding the absence of a photon as logical "0" while its presence would correspond to logical "1".

The advantage of our scheme over other conventional sources of single-photon wavepackets such as down-conversion is twofold. First of all, our scheme offers excellent control over the instances in time when a qubit is created as well as over the spectral composition of the wavepacket. The qubits may thus be generated with a well defined *tact frequency*. Secondly, the coherent recycling of the state of the active medium after the generation of a qubit gives rise to entanglement between subsequent qubits. In particular n-qubit maximally entangled states (MES), e.g., GHZ-states [11] for $n = 3$, can be conveniently generated in this way.

The qubits produced by our scheme can be utilized in various quantum cryptography [12] and teleportation [13] schemes, as well as in tests of non-locality and multiparticle interference [14]. Because the production of MES of n qubits is straightforward, our scheme is well suited for quantum communication between three or more parties. Several recent proposals for n-users quantum cryptography key distribution [15], quantum secret sharing [16], cryptographic conferencing, and multiparticle generalisations of superdense coding [17] assume the parties involved to share n-qubit maximally entangled states. In a future quantum network our scheme could be used by a trusted provider to generate entangled states on demand and distribute them to clients through a network with starshaped topology. Subsequent communication between the recipients who now share an entangled state only needs to take place classically thus freeing them from the need to maintain separate quantum channels between each other.

In Ref. [7] we introduced our proposal by considering a specific type of active medium, namely a trapped and externally driven atom/ion inside an optical resonator. Here we favour a model independent formulation since other implementations based on quantum dots [18] or molecules [19, 20] may emerge in the course of time. In section 2 we give a general description of the requirements for both the source and type of interaction necessary to warrant entangled photon wavepacket generation. We also show how the physical creation process may be represented in terms of abstract quantum circuit diagrams. The general formalism is presented in section 3. In section 4 we demonstrate that if there is no significant temporal overlap between successively generated photons we may regard the multiphoton state as a product state of independent entities. This is essential for an interpretation of the individual wavepackets as independent logical quantum bits. Finally, we discuss the effect of decoherence in this system and show that up to ten maximally entangled qubits can be generated with a fidelity of at least 90%. In section 6 we summarize our findings and give a brief discussion of possible applications of multi-qubit entangled states in quantum communication.

2 Motivation

The most promising proposal so far for a source of well resolved 1-photon wavepackets with an adjustable spectral envelope has been the *photon gun* proposed by Law and Kimble [21, 22].

Their proposal differs from previously made ones [18–20] insofar as it offers perfect control over the pulse shape as well as the time window T during which the wavepacket is created. Several quantum information protocols [3] which so far have to rely on the use of attenuated coherent fields would benefit from such a cleaner source. For other purposes in future quantum networks the source proposed in [21] may not be adequate since it lacks one key ingredient that is *entanglement* between subsequently created photons.

With multi-party quantum communication in mind the need for a source of *entangled* photonic qubits becomes immediately obvious. The scheme we propose here employs conditional wavepacket creation. This means that the polarization of the created wavepacket depends on the internal state of the medium. The encoding of quantum information in the photonic qubit is accomplished most conveniently using two orthogonal polarization states of the field. Thus we have to consider a medium which reflects this fact: in our proposal it has two possible initial states each giving rise to a photon with a different polarization state. If the medium is initially in a superposition of these two states we may create entangled medium-field states. An algorithm for the creation of entangled states would thus read:

(0) Prepare initially the medium in a state $|\Psi\rangle_m = c_0|i_0\rangle + c_1|i_1\rangle$, here $|i_0\rangle$ and $|i_1\rangle$ span the medium qubit.

(1) Find an interaction which maps each medium state $|i_\alpha\rangle$ to a unique ancilla state $|f_\alpha\rangle$ thereby creating a single photon with *logical* state $|\alpha\rangle_p$, i.e., after some time T: $|i_\alpha\rangle|\text{vacuum}\rangle \to |f_\alpha\rangle|\alpha\rangle_p$. Here p denotes photon.

(2) Next recycle the medium: $|f_\alpha\rangle \to |i_\alpha\rangle$. We have created an entangled state $|\Psi'\rangle = c_0|i_0\rangle|0\rangle_p + c_1|i_1\rangle|1\rangle_p$ in the process.

(3) Alter the state of the medium as desired by a 1-qubit rotation of the states $|i_\alpha\rangle$ and repeat steps **(1)–(3)**. This way we are able to create higher order entangled medium-field states.

3 The model

In this section we will develop a theory to describe the wavepacket generation in a rather general context. We are not concerned with any specific model for the active medium, e.g., a single trapped atom/ion inside the resonator, and we first consider interactions of the medium with a single (polarization) mode only. These simplifications will be abandoned in section 4 when we establish the interpretation of 1-photon wavepackets as qubits. The Hamiltonian reads as follows (in the standard dipole and rotating wave approximations):

$$H/\hbar = H_0 + V, \quad H_0 = \omega_c a^\dagger a + \int d\omega\, \omega b^\dagger(\omega) b(\omega) + H_\mathrm{m} \tag{1}$$

$$V = i\int d\omega k(\omega)\left(a b^\dagger(\omega) - b(\omega) a^\dagger\right) + \sum_j \theta(t-t_j)\theta(t_j+T-t) V_\mathrm{mc}(t), \tag{2}$$

where ω_c denotes the cavity frequency; $a^\dagger$, $b^\dagger(\omega)$ (a, $b(\omega)$) are the creation (annihilation) operators of the cavity and reservoir modes, respectively. The reservoir modes satisfy standard bosonic commutation relations: $[b(\omega), b^\dagger(\nu)] = \delta(\omega-\nu)$. The coupling between the cavity and the reservoir modes is described by $k(\omega)$. Assuming the coupling to be flat around the cavity resonance frequency (i.e., adopting the first Markov approximation) we may set $k(\omega) \approx \sqrt{\kappa_c/\pi}$ [23]. The term H_0 contains the free dynamics with H_m denoting the free

Hamiltonian of the active medium. Finally, $V_{\rm mc}$ describes the interaction between the cavity mode and the active medium. We assume that the (repeated) creation of a single photon inside the cavity takes place within a time window of duration T starting at times t_j, with the condition that $t_{j+1}-t_j > T$. The Heaviside step functions employed in Eq. (2) serve the purpose of expressing in a convenient fashion the limited duration T during which the interaction in each wavepacket generation sequence is switched on; note that we assume that $V_{\rm mc}$ vanishes smoothly at the endpoints of each time window. In an interaction picture with respect to the free dynamics given by H_0 we have

$$V_I = i\sqrt{\frac{\kappa_c}{\pi}}\int d\omega\left(ab^\dagger(\bar\omega)e^{i\omega t} - b(\bar\omega)e^{-i\omega t}a^\dagger\right) + V^I_{\rm mc}(t), \tag{3}$$

where $\bar\omega \equiv \omega+\omega_c$ and $V^I_{\rm mc}(t) = \sum_j \theta(t-t_j)\theta(t_j+T-t)e^{iH_0t}V_{\rm mc}(t)e^{-iH_0t}$. The evolution operator corresponding to the medium-cavity interaction is defined as:

$$U_{\rm mc}(t) = \mathcal{T}\exp(-i\int_0^t dt' V^I_{\rm mc}(t')),$$

where $\mathcal{T}$ denotes temporal ordering. The family of interactions that we wish to consider may be characterised by two requirements:

(i) the interaction transforms a given initial (metastable) state of the medium into a known final (metastable) state: $|i\rangle_{\rm m} \to |f\rangle_{\rm m}$.

(ii) in the course of the interaction *at most* one photon is deposited in the cavity mode. The interaction is slow on the timescale given by the cavity lifetime κ_c^{-1}, i.e. $T \gg \kappa_c^{-1}$.

We assume both the cavity and the reservoir to be initially in the vacuum state. Then, endowed with the above mentioned premisses the evolution for $t < t_1$ is restricted to a subspace with only one photonic excitation present. Thus the following ansatz is appropriate for the wavefunction evolution

$$|\psi(t)\rangle = |\varphi(t)\rangle_{\rm mc}|0\rangle_r + \int d\omega|\varphi_{\bar\omega}(t)\rangle_{\rm mc}|1_{\bar\omega}\rangle_{\rm r}, \quad \text{with} \quad |\varphi(0)\rangle_{\rm mc} = |i\rangle_{\rm m}|0\rangle_{\rm c}. \tag{4}$$

Here $|\varphi_{\bar\omega}(t)\rangle_{\rm mc}$ and $|\varphi(t)\rangle_{\rm mc}$ denote medium-cavity states with and without the transfer of a photon having taken place into the reservoir mode with frequency $\bar\omega$, respectively. Their behavior is governed by the following equations:

$$\begin{aligned} |\dot\varphi(t)\rangle_{\rm mc} &= -iV^I_{\rm mc}(t)|\varphi(t)\rangle_{\rm mc} - \sqrt{\frac{\kappa_c}{\pi}}\int d\omega e^{-i\omega t}a^\dagger|\varphi_{\bar\omega}(t)\rangle_{\rm mc} \\ |\dot\varphi_{\bar\omega}(t)\rangle_{\rm mc} &= -iV^I_{\rm mc}(t)|\varphi_{\bar\omega}(t)\rangle_{\rm mc} + \sqrt{\frac{\kappa_c}{\pi}}e^{i\omega t}a|\varphi(t)\rangle_{\rm mc}, \end{aligned}$$

Formal integration of the evolution equation for $|\varphi_{\bar\omega}(t)\rangle_{\rm mc}$ yields

$$|\varphi_{\bar\omega}(t)\rangle_{\rm mc} = \sqrt{\frac{\kappa_c}{\pi}}\int_0^t dt' U_{\rm mc}(t,t')e^{i\omega t'}a|\varphi(t')\rangle_{\rm mc}, \tag{5}$$

where $U_{\rm mc}(t,t') = U_{\rm mc}(t)U^\dagger_{\rm mc}(t')$. Insertion of Eq. (5) into the evolution equation of $|\varphi(t)\rangle_{\rm mc}$ yields, within the validity of the Markov-approximation [24],

$$|\varphi(t)\rangle_{\rm mc} = U^{\rm eff}_{\rm mcr}(t)|\varphi(0)\rangle_{\rm mc}, \text{with} \quad U^{\rm eff}_{\rm mcr} = \mathcal{T}\exp(-i\int_0^t dt'[V^I_{\rm mc}(t') - i\kappa_c a^\dagger a]) \tag{6}$$

is the effective evolution operator generated by a non-Hermitian Hamiltonian describing both the medium-cavity interaction and the decay into the reservoir. Hence we obtain

$$|\varphi_{\bar{\omega}}(t)\rangle_{\rm mc} = \sqrt{\frac{\kappa_c}{\pi}} \int_0^t dt' e^{i\omega t'} U_{\rm mc}(t,t') a U_{\rm mcr}^{\rm eff}(t') |\varphi(0)\rangle_{\rm mc}. \tag{7}$$

Since we are interested in the state of the reservoir after the photon has left the resonator with near certainty we only consider times t exceeding T by a few cavity lifetimes. The contribution to the integral from times larger than T vanishes since there $U_{\rm mcr}^{\rm eff}(t)$ represents merely an exponential decay, i.e.,

$$|\varphi(t)\rangle_{\rm mc} = \exp(-\kappa_c a^\dagger a(t-T))|\varphi(T)\rangle_{\rm mc} \tag{8}$$

Any 1-photon component of $|\varphi(t)\rangle_{\rm mc}$ will thus be strongly suppressed. The assumption of efficient transfer **(i)** guarantees that the projection of $|\varphi(T)\rangle_{\rm mc}$ on the vacuum component is insignificant. We may thus assume that the contribution of state $|\varphi(t)\rangle_{\rm mc}|0\rangle_r$ to the total state $|\psi(t)\rangle$ vanishes. We may therefore write

$$|\psi(t)\rangle = \sqrt{\frac{\kappa_c}{\pi}} \int \int_0^T d\omega dt' e^{i\omega t'} U_{\rm mc}(t,t') a U_{\rm mcr}^{\rm eff}(t') |\varphi(0)\rangle_{\rm mc} |1_{\bar{\omega}}\rangle_{\rm r}. \tag{9}$$

According to the requirement **(i)** the interaction with the medium must guarantee an efficient transfer of a single photon to the resonator within its duration T, i.e. it will connect the initial state $|i\rangle_{\rm m}|0\rangle_c$ only to the state $|f\rangle_m|1\rangle_{\rm c}$. Consequently, the action of the term $U_{\rm mc}(t,t')$ in the integral kernel on the right hand side of Eq. (9) will be twofold. First it will erase the state component proportional to $|i\rangle_{\rm m}|0\rangle_c$ and transform $|f\rangle_m|1\rangle_{\rm c}$ into $|f\rangle_m|0\rangle_{\rm c}$. The latter is not affected by the interaction thus $U_{\rm mc}(t,t')$ leaves it unaltered. In formal terms this can be rephrased as: $V_{\rm mc}(t)|f\rangle_{\rm m}|0\rangle_c = 0$. Physically, this means that the generation of photons stops after the first generated photon has left the cavity. We may then recast Eq. (9) in the following simpler form

$$|\psi(t)\rangle = \int d\omega a G(\omega,T)|\varphi(0)\rangle_{\rm mc}|1_{\bar{\omega}}\rangle_{\rm r}, \tag{10}$$

where

$$G(\omega,T) \equiv g(\omega,T) a^\dagger \sigma_{fi} = \sqrt{\frac{\kappa_c}{\pi}} \int_0^T dt e^{i\omega t} U_{\rm mcr}^{\rm eff}(t). \tag{11}$$

The operator σ_{fi} maps the state $|i\rangle_{\rm m}$ to the state $|f\rangle_{\rm m}$. The spectral envelope of the created wavepacket is given by $g(\omega,T)$ (a mere c-number function). Conservation of the norm of $|\psi\rangle$ implies $\int d\omega |g(\omega,T)|^2 = 1$.

We may now recycle the medium back to its initial state $|i\rangle_m$ and repeat the generation anew starting at some time $t_1 > T$. For times larger than $t_1 + T$ we have then created a two-photon wavepacket with no significant temporal overlap of the individual photons. After a few lines of algebra one obtains the following approximate result for the state for times $t > t_1 + T$

$$|\psi(t)\rangle = \int d\omega d\nu \, aG(\nu,T) e^{i\nu t_1} \sigma_{if} a G(\omega,T) |\varphi(0)\rangle_{\rm mc} |1_{\bar{\omega}} 1_{\bar{\nu}}\rangle_{\rm r}. \tag{12}$$

4 Photon wavepackets as qubits

Any physical realization of logical qubits is contingent upon the availability of distinguishable and independently controllable (two-state) quantum systems which interact with each other only in a controlled manner. In other words, the annihilation and creation operators corresponding to different qubits must commute. We will now show that within the framework of the present theory we may identify such approximately commuting creation and annihilation operators with the multimode operators that create (annihilate) the individual wavepackets. This thus allows us to regard the wavepackets as independent quantum entities provided there is no appreciable temporal overlap between successively generated wavepackets.

Factorized photon wavepackets: Let us assume that the time between subsequent photon generation processes is so long that the previous photon has leaked out of the cavity with almost certainty before the next photon is produced. Then we can generalize the result for the state of two photonic qubits as given by Eq. (12) to the case of n qubits, and formally express the state as a product of multimode creation operators applied to the vacuum of the continuum. The n-photon wavepacket state can be written as

$$|\psi\rangle_{\mathrm{r}} = \prod_{j=0}^{n} B^{\dagger}(t_j, T)|0\rangle_{\mathrm{r}}, \quad \text{with} \quad B^{\dagger}(t_j, T) = \int d\omega\, e^{i\omega t_j} g(T, \omega) b^{\dagger}(\bar{\omega}). \tag{13}$$

and $t_0 = 0$. In order to arrive at a description of the n-photon wavepacket in terms of a product of n photonic qubits we have to show that $[B(t_j, T), B^{\dagger}(t_k, T)] = \delta_{jk}$ holds to a very good approximation. Inserting Eq. (13) yields

$$[B(t_j, T), B^{\dagger}(t_k, T)] = \int d\omega e^{i\omega(t_k - t_j)} |g(\omega, T)|^2.$$

It is plausible to assume that the mode distribution function $|g(\omega)|^2$ is a reasonably well behaved smooth function which may be assigned something like a half width at half maximum. Let us refer to this quantity as $\delta\omega < \kappa_c$. Its inverse may by associated with the pulse duration $\tau = 1/\delta\omega$. The Fourier transform of $|g(\omega, T)|^2$ will thus be approximately given by an expression of the following type

$$\int d\omega e^{i\omega(t_j - t_k)} |g(\omega, T)|^2 \approx \left(\exp(-(|t_j - t_k|/\tau)^{\rho})\right)^{\xi}, \tag{14}$$

with $\xi > 0$ and $\rho \geq 1$ (the latter only holds provided the spectral density drops off asymptotically at least as fast as ω^{-2} which may be assumed since the distribution is limited in width by the Lorentzian lineshape of the resonator). We realise that the rhs of Eq. (14) becomes extremely small if $|t_j - t_k| \gg \tau$, that is if there is no overlap between successively generated wavepackets. In this case we may regard each of the $B(t_j, T)$ as acting on its own vacuum. Physically, this corresponds to the fact that the individual wavepacket can be thought of as being localized each within a box of length cT, all of which do not overlap and travel at the speed of light. So we can rewrite Eq. (13) as

$$|\psi\rangle_{\mathrm{r}} = \prod_{j=0}^{n} |\psi\rangle_j, \quad |\psi\rangle_j = B^{\dagger}(t_j, T)|0\rangle_{\mathrm{r_j}}. \tag{15}$$

This paves the way for an interpretation of the wavepackets as distinguishable quantum entities.

Logical qubit states with polarized photons: Let us now make use of both polarization states (e.g., σ_+, σ_-) of a single longitudinal mode of an optical resonator. Formally, we will thus have to consider two independent resonator modes a_α, $\alpha = 0, 1$. In order to warrant clean conditional wavepacket generation the medium-cavity interaction must not contain any polarization mixing terms. As indicated in the motivational section the medium must be prepared in a superposition state. Let us denote by $|i_\alpha\rangle_m$, $|f_\alpha\rangle_m$ ($\alpha = 0, 1$) the states involved in the creation of a σ_+ (0) and σ_- (1) polarized photon, respectively. In the absence of polarization mixing terms the subsystems corresponding to different values of α are completely decoupled and we may expect the scheme to implement the following mapping:

$$|i_\alpha\rangle_{\mathrm{m}}|0\rangle_c \longrightarrow a_\alpha^\dagger |f_\alpha\rangle_{\mathrm{m}}|0\rangle_c, \qquad \alpha = 0, 1.$$

If we assume the medium to be initially prepared in a superposition state $|\varphi(0)\rangle_m = c_0|i_0\rangle + c_1|i_1\rangle$ then the final state of the total system can be directly obtained from the superposition principle by issuing each partial solution with the appropriate probability amplitude c_α. We can define wavepacket creation operators like in Eq. (15) for both polarizations, $B_0^\dagger(t_j, T)$ and $B_1^\dagger(t_j, T)$, which are composed of the single frequency creation operators $b_0^\dagger(\bar\omega)$ and $b_1^\dagger(\bar\omega)$. We are thus able to identify the one-photon wavepackets as qubits with logical states $|\alpha\rangle_j \equiv B_\alpha^\dagger(t_j, T)|0\rangle_r$, where $\alpha = 0, 1$ refers to the logical (polarization) state. We have thus shown that it is permissible to regard the n-photon pulse generated by our scheme as an array of n qubits in an entangled state with the logical state being assoociated with the polarization state of each 1-photon wavepacket the pulse is made up of.

Quantum state engineering: Logical states of the multi-qubit system are created by preparing the active medium initially in a superposition state:

$$|\psi(0)\rangle = (\cos\phi_1 |i_0\rangle_m + \sin\phi_1 e^{i\varphi_1}|i_1\rangle_m)|0\rangle_c|0\rangle_r.$$

This is accomplished by connecting levels $|i_0\rangle$ and $|i_1\rangle$ through an appropriate interaction mechanism, e.g., in the case of the medium being an atom, a Raman laser pulse or a microwave field of appropriate duration and detuning is applied to the transition. Such a 1-qubit rotation is realized by a unitary transformation parametrized by the angles ϕ_1 and φ_1. After the generation of the first photon the state results

$$(\cos\phi_1|f_0\rangle_m|0\rangle_1 + \sin\phi_1 e^{i\varphi_1}|f_1\rangle_m|1\rangle_1)|0\rangle_c.$$

We can now recycle the states $|f_\alpha\rangle_m$ back to $|i_\alpha\rangle_m$ and repeat the procedure, with angles ϕ_2 and φ_2, and similarly for all subsequently generated photons. In this way we are able to generate various logical states of the n-qubit system. In our scheme we have only $2n$ independent coeffiecients at hand, thus the size of the class of states that can be prepared is limited by this fact. Eventually the residual entanglement with the medium can be removed by measuring the medium state in an appropriate basis. For example, in the three-photon case, if we choose $\phi_1 = \pi/4, \phi_2 = \phi_3 = \varphi_1 = \varphi_2 = \varphi_3 = 0$, disregarding a normalization constant the final state is $(|f_0\rangle_m|0\rangle_1|0\rangle_2|0\rangle_3 - |f_1\rangle_m|1\rangle_1|1\rangle_2|1\rangle_3)|0\rangle_c$. By measuring the atom in the basis $|f_0\rangle_m \pm |f_1\rangle_m$ we can remove the entanglement with the medium and obtain one of the two GHZ-states $|000\rangle \mp |111\rangle$; the preparation of a higher dimentional MES $|\Psi\rangle = \frac{1}{\sqrt{2}}(|s_1, s_2, ..., s_n\rangle + |1 - s_1, 1 - s_2, ..., 1 - s_n\rangle)$, $s_j = 0, 1$, is analogous.

5 Decoherence

A rough estimate of the qubit generation rate our scheme would seem capable of is given by the inverse of the duration of a generation/preparation sequence. In practice the maximum number n of entangled photon wavepackets (qubits) that our scheme can generate with a certain fidelity is limited by technical imperfections, atomic spontaneous emission, and random loss of photons inside the output coupler (cavity mirror). Below we will analyse the impact of each of these perturbations on the reliability with which a certain state can be generated from the point of view of the model just considered.

Spontaneous emission: Spontaneous emission of the atoms making up the medium can be quenched by operating the device sufficiently far off resonance. For the experimental parameters used in Ref. [7] we estimated the probability of a spontaneous emission per cycle to be of the order of 10^{-3}.

Absorption in the cavity mirrors: Photon absorption in the cavity mirrors is an essential effect for high–Q optical cavities. In general, it leads to two types of errors: Photon absorption and concomitant destruction of the entanglement. The first type of errors are evaded by using post-selection schemes, which correspond here to discarding sequences with a number of detected photons smaller than n. State distortion [25] can occur even in the absence of loss of a photon if the absorption rates for both polarisation are unequal $\kappa_1 \neq \kappa_0$. This effect can be kept small for a cavity with equal absorption rates for both polarizations, i.e., $\kappa_0 \simeq \kappa_1$.

Imperfections in the recycling of states: Rather unexpectedly, the most prominent source of decoherence is due to imperfections in the recycling between the ground states $|f_\alpha\rangle \to |i_\alpha\rangle$. The required π-pulses could be ill timed or their effect influenced by the motion of the atom. Moreover, there could be a dephasing between the probability amplitudes of the states $|f_\alpha\rangle$ of the two Λ-systems due to laser fluctuations in the Raman transitions. To assess their importance we assume the following imperfect mapping in each of the generation sequences: $|f_\alpha\rangle \to A_\alpha|i_\alpha\rangle + B_\alpha|f_\alpha\rangle$, where $A_\alpha = (1-\epsilon_\alpha)\exp(i\delta_\alpha)$. When preparing an n-qubit MES the resulting state is, for fixed A_α^j, of the form $|\Psi\rangle = (A_0^1 A_0^2 ... A_0^n|00...0\rangle + A_1^1 A_1^2 ... A_1^n|11...1\rangle + \text{others})/\sqrt{2}$, where "others" refers to states containing less than n photons (qubits). We assume that the magnitude of ϵ_α is evenly distributed over a range of $[0, \epsilon_m]$. The dephasing angle is evenly distributed over $[-\delta_m, \delta_m]\pi$. The final density matrix is given as an average over different realizations, $\rho = \text{Average}[|\Psi\rangle\langle\Psi|]$. In Fig. 1 we plot the fidelity $\mathcal{F}(n)$ of an n-qubit MES produced by our model source. The fidelity is defined as

$$\begin{aligned}\mathcal{F}(n) &= \frac{1}{2}(\langle 00...0| + \langle 11...1|)\rho(|00...0\rangle| + |11...1\rangle) \qquad (16)\\ &= \frac{1}{4}\text{Average}[|A_0^1 A_0^2 ... A_0^n|^2 + |A_1^1 A_1^2 ... A_1^n|^2 + 2\text{Re}(A_0^1 A_0^2 ... A_0^n (A_1^1 A_1^2 ... A_1^n)^*)].\end{aligned}$$

and represents the average overlap between the desired n-photon wavepacket and the one resulting from an imperfect sequence of recycling steps. Fig. 1 shows that the process is rather robust against global dephasing (δ_α) but that the correct timing of the π-pulses is critical. From curve (a) we gather that for errors in the 2% range approximately 10 qubits can be created with a fidelity of 90%. The fidelity $\mathcal{F}(n)$ drops off exponentially with the number of qubits, as can be seen from a simple analytical estimate in the case of Gaussian-distributed error. Let us consider for simplicity the case $\delta_\alpha^j = 0$. We assume that the width σ of the error distribution is so small

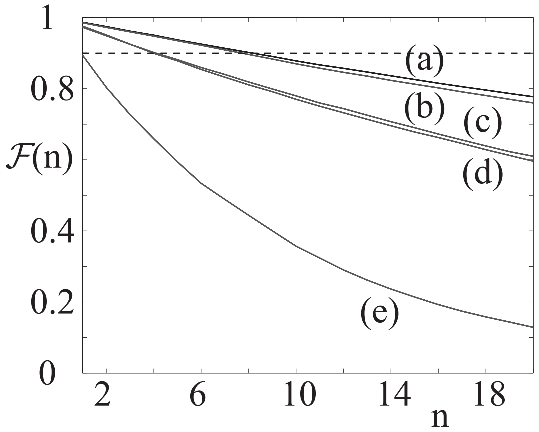

Fig. 1. Ensemble averaged fidelity as a function of the number of qubits n. Curves (a), (c), and (e) assume $\delta_m = 0$ and $\epsilon_m = 0.0125, 0.025, 0.1$, respectively. Curves (b) and (d) are the same as (a) and (c) with $\delta_m = 0.05$.

that we can extend the upper limit $\epsilon_\alpha^j = 1$ in the integrations into infinity. For $\sigma \ll 1$ this yields $\mathcal{F}(n) \simeq [1 - 2\sigma/\sqrt{\pi}]^n$. For large n we obtain the exponential decay $\mathcal{F}(n) \sim \exp(-2\sigma n/\sqrt{\pi})$. In accordance with the simulation, $\sigma = 0.0125$ gives $\mathcal{F}(10) \sim 0.87$.

6 Summary

We have presented a general proposal for the controlled generation of entangled n-qubit states, where non-overlapping one-photon wavepackets are identified with the logical qubits. The theory developed is general although a CQED based implementation seems most promising from the present point of view. However, it can be easily adapted to other implementations, for instance quantum dots or single atoms embedded in a host material [18] which may become available in the future as quantum systems with long coherence times. The logical qubit states are identified with two orthogonal transverse polarization states (e.g., $\sigma_{+/-}$) of the cavity and reservoir modes. Our investigation of decoherence in this model indicates that such a proposal is in principle experimentally doable with state-of-the-art equipment. It could thus form the experimental basis for multi-party communication in future quantum networks. Of particular interest are several recent proposals [16, 17] which have suggested to make use of many-qubit entangled states in order to share quantum information between several parties. Another interesting application is a controlled quantum communciation network, which consists of a single “provider” (P) who is connected to the clients (A-D) via photonic channels [8]. The provider creates entangled photonic qubits on request and transmits them to the clients. Upon transmission all three parties involved share an entangled 3-qubit state. In order for A or B to teleport the state of an additional qubit between each other the central station P has to measure the state of its qubit (which would be the internal state of the medium) and disclose the result to A and B. The advantage of such a scheme is its relatively simple topology which eliminates the need for photonic channels between the individual nodes A-D. Moreover, no successful transfer of information is possible without the consent of the provider. We have thus a network in which the transfer of information between two parties is always controlled by a third one. This may be of practical importance if the time at which information transfer takes place should be determined by the third party.

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