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DECAY CONTROL IN DISSIPATIVE QUANTUM SYSTEMS*

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We point out that the quantum Zeno effect, i.e., inhibition of spontaneous decay by frequent measurements, is observable only in spectrally finite reservoirs, i.e., in cavities and waveguides, using a sequence of evolution-interrupting pulses or randomly-modulated CW fields. By contrast, such measurements can only accelerate decay in free space.

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1 Introduction

The "watchdog" or quantum Zeno effect (QZE) is a basic manifestation of the influence of measurements on the evolution of a quantum system. The original QZE prediction has been that *irreversible decay* of an excited state into an open-space reservoir can be inhibited [1], by repeated interruption of the system-reservoir coupling, which is associated with measurements (e.g., the interaction of an unstable particle with its environment on its flight through a bubble chamber) [2, 3]. However, this prediction has not been experimentally verified as yet! Instead, the interruption of Rabi oscillations and analogous forms of *nearly-reversible* evolution has been at the focus of interest [4–11]. Tacit assumptions have been made that the QZE is in principle attainable in open space, but is technically difficult.

We have recently demonstrated [12] that the inhibition of *nearly-exponential* excited-state decay by the QZE in two-level atoms, in the spirit of the original suggestion [1], is amenable to experimental verification in resonators. Although this task has been widely believed to be very difficult, we have shown, by means of our unified theory of spontaneous emission into arbitrary reservoirs [13], that two-level emitters in cavities or in waveguides are in fact adequate for radiative decay control by the QZE [12]. Condensed media or multi-ion traps are their analogs for vibrational decay control (phonon emission) by the QZE [14]. We have now developed a more comprehensive view of the possibilities of excited-state decay by QZE. Here we wish to demonstrate that QZE is indeed achievable by repeated or continuous measurements of the excited state, but only in reservoirs whose spectral response *rises up* to a frequency which does not exceed the resonance (transition) frequency. By contrast, in open-space decay, where the reservoir response has a much higher cutoff, *non-destructive* frequent measurements are much more likely to accelerate decay, causing the anti-Zeno effect.

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2 Measurement schemes

2.1 Impulsive measurements (Cook's scheme)

Consider an initially excited two-level atom coupled to an *arbitrary* density-of-modes (DOM) spectrum $\rho(\omega)$ of the electromagnetic field in the vacuum state. At time τ its evolution is interrupted by a short optical pulse, which serves as an impulsive quantum measurement [4–11]. Its role is *to break the evolution coherence*, by transferring the populations of the excited state $|e\rangle$ to an auxiliary state $|u\rangle$ which then decays back to $|e\rangle$ *incoherently*.

The spectral response, i.e., the emission rate into this reservoir at frequency ω, is

$$G(\omega) = |g(\omega)|^2 \rho(\omega), \tag{1}$$

$\hbar g(\omega)$ being the field-atom coupling energy.

We cast the excited-state amplitude in the form $\alpha_e(\tau)e^{-i\omega_a\tau}$, where ω_a is the atomic resonance frequency. Restricting ourselves to sufficiently short interruption intervals τ such that $\alpha_e(\tau) \simeq 1$, yet long enough to allow the rotating wave approximation, we obtain

$$\alpha_e(\tau) \quad \simeq \quad 1 - \int_0^\tau dt(\tau - t)\Phi(t)e^{i\Delta t}, \tag{2}$$

where

$$\Phi(t) = \int_0^\infty d\omega G(\omega) e^{-i(\omega-\omega_s)t}. \tag{3}$$

$\Delta = \omega_a - \omega_s$ is the detuning of the atomic resonance from the peak (or cutoff) ω_s of $G(\omega)$.

To first order in the atom-field interaction, the excited state probability after n interruptions (measurements), $W(t = n\tau) = |\alpha_e(\tau)|^{2n}$, can be written as

$$W(t = n\tau) \approx [2\mathrm{Re}\alpha_e(\tau) - 1]^n \approx e^{-\kappa t}, \tag{4}$$

where

$$\kappa = \frac{2}{\tau}\mathrm{Re}[1 - \alpha_e(\tau)] = \frac{2}{\tau}\mathrm{Re}\int_0^\tau dt(\tau - t)\Phi(t)e^{i\Delta t}. \tag{5}$$

The QZE obtains if κ *decreases with* τ for sufficiently short τ. This essentially means that the correlation (or memory) time of the field reservoir is longer (or, equivalently, $\Phi(t)$ falls off slower) than the chosen interruption interval τ.

Equation (5) can be rewritten as

$$\kappa = 2\pi \int G(\omega) \left\{ \frac{\tau}{2\pi} \mathrm{sinc}^2 \left[\frac{(\omega - \omega_a)\tau}{2} \right] \right\} d\omega, \tag{6}$$

where the interruptions are seen to cause dephasing whose spectral width is $\sim 1/\tau$.

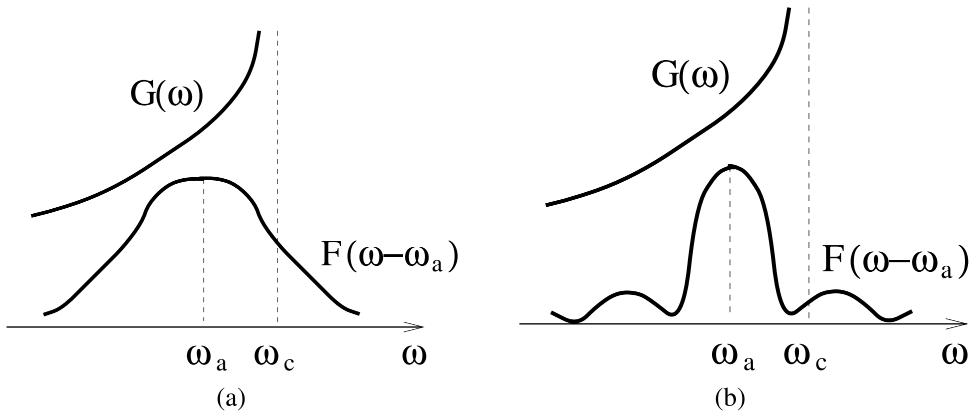

Fig. 1. Dependence of effective decay rate κ on dephasing (relaxation) spectrum $F(\Delta)$ and field reservoir response with cutoff $G(\omega)$: (a) Lorentzian dephasing spectrum [Eq. (7)]; (b) sinc-function spectrum [Eq. (6) – impulsive measurements].

2.2 Noisy-field dephasing: Random Stark shifts

Instead of disrupting the coherence of the evolution by a sequence of "impulsive" measurements, as above, we can achieve this goal by *noisy-field dephasing* of $\alpha_e(t)$: Random ac-Stark shifts by an off-resonant intensity-fluctuating field result in the replacement of Eq. (6) by (Fig. 1)

$$\kappa = \int G(\Delta + \omega_a)\mathcal{L}(\Delta)d\Delta. \tag{7}$$

Here the spectral response $G(\Delta+\omega_a)$ is the same as in Eq. (1), whereas $\mathcal{L}(\Delta)$ is the Lorentzian-shaped relaxation function of the coherence element $\rho_{eg}(t)$, which for the common dephasing model decays *exponentially*. This Lorentzian relaxation spectrum has a HWHM width $\nu = \langle\Delta\omega^2\rangle\tau_c$, the product of the mean-square Stark shift and the noisy-field correlation time. The QZE condition is that *this width be larger than the width of* $G_s(\omega)$ (Fig. 1). The advantage of this realization is that it does not depend on γ_u, and is realizable for any atomic transition. Its importance for molecules is even greater: if we start with a single vibrational level of $|e\rangle$, no additional levels will be populated by this process.

2.3 CW dephasing

The random ac-Stark shifts described above cause both shifting and broadening of the spectral transition. If we wish to avoid the shifting altogether, we may employ a CW driving field that is *nearly resonant* with the $|e\rangle \leftrightarrow |u\rangle$ transition [4, 5]. If the decay rate of this transition, γ_u, is larger than the Rabi frequency Ω of the driving field, then one can show that κ is given again by Eq. (7), where the Lorentzian (dephasing) width is

$$\nu = \frac{2\Omega^2}{\gamma_u}. \tag{8}$$

2.4 Universal formula

All of the above schemes are seen to yield the same universal formula for the decay rate

$$\kappa = 2\pi \int G(\omega)F(\omega - \omega_a)d\omega, \tag{9}$$

where $F(\omega)$ expresses the relevant measurement-induced dephasing (sinc- or a Lorentzian-shaped): its width relative to that of $G(\omega)$ determines the QZE behavior.

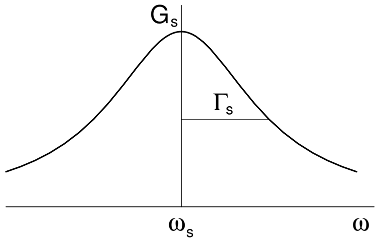

Fig. 2. Cavity mode with Lorentzian lineshape.

3 Applications to various reservoirs

3.1 Finite reservoirs: A Lorentzian line

The simplest application of the above analysis is to the case of a two-level atom coupled to a near-resonant Lorentzian line centered at ω_s, characterizing a high-Q cavity mode [12]. In this case,

$$G_s(\omega) = \frac{g_s^2\Gamma_s}{\pi[\Gamma_s^2 + (\omega - \omega_s)^2]}, \tag{10}$$

where g_s is the resonant coupling strength and Γ_s is the linewidth (Fig. 2). Here $G_s(\omega)$ stands for the sharply-varying (nearly-singular) part of the DOM distribution, associated with narrow cavity-mode lines or with the frequency cutoff in waveguides or photonic band edges. The broad portion of the DOM distribution $G_b(\omega)$ (the "background" modes), always coincides with the free-space DOM $\rho(\omega) \sim \omega^2$ at frequencies well above the sharp spectral features. In an open cavity, $G_b(\omega)$ represents the atom coupling to the unconfined free-space modes. This gives rise to an *exponential* decay factor in the excited state probability, regardless of how short τ is, i.e.,

$$\kappa = \kappa_s + \gamma_b, \tag{11}$$

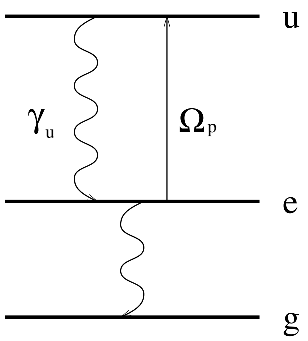

Fig. 3. Cook's scheme for impulsive measurements.

where κ_s is the contribution to κ from the sharply-varying modes and $\gamma_b = 2\pi G_b(\omega_a)$ is the effective rate of spontaneous emission into the background modes. In most structures γ_b is comparable to the free-space decay rate γ_f.

In the short-time approximation, taking into account that the Fourier transform of the Lorentzian $G_s(\omega)$ is $\Phi_s(t) = g_s^2 e^{-\Gamma_s t}$, Eq. (2) yields (without the background-modes contribution)

$$\alpha_e(\tau) \approx 1 - \frac{g_s^2}{\Gamma_s - i\Delta}\left[\tau + \frac{e^{(i\Delta-\Gamma_s)\tau} - 1}{\Gamma_s - i\Delta}\right]. \tag{12}$$

The QZE condition is then

$$\tau \ll (\Gamma_s + |\Delta|)^{-1}, g_s^{-1}. \tag{13}$$

On resonance, when $\Delta = 0$, Eqs. (5) and (12) yield

$$\kappa_s = g_s^2 \tau. \tag{14}$$

Thus the background-DOM effect cannot be modified by QZE. Only the sharply-varying DOM contribution κ_s may allow for QZE. Only the κ_s term decreases with τ, indicating the QZE inhibition of the nearly-exponential decay into the Lorentzian field reservoir as $\tau \to 0$. Since Γ_s has dropped out of Eq. (14), the decay rate κ is the *same* for both strong-coupling ($g_s > \Gamma_s$) and weak-coupling ($g_s \ll \Gamma_s$) regimes. Physically, this comes about since for $\tau \ll g_s^{-1}$ the energy uncertainty of the emitted photon is too large to distinguish between reversible and irreversible evolutions.

The evolution inhibition, however, has rather different meaning for the two regimes. In the weak-coupling regime, where, in the absence of the external control, the excited-state population decays nearly exponentially at the rate $g_s^2/\Gamma_s + \gamma_b$ (at $\Delta = 0$), one can speak about the inhibition of irreversible decay, in the spirit of the original QZE prediction [1]. By contrast, in the strong-coupling regime in the absence of interruptions (measurements), the excited-state population undergoes damped Rabi oscillations at the frequency $2g_s$. In this case, the QZE slows down the evolution during the first Rabi half-cycle ($0 \le t \le \pi/2g_s^{-1}$), the evolution on the whole becoming irreversible.

A possible realization of this scheme is as follows. Within an open cavity the atoms repeatedly interact with a pump laser, which is resonant with the $|e\rangle \to |u\rangle$ transition frequency.

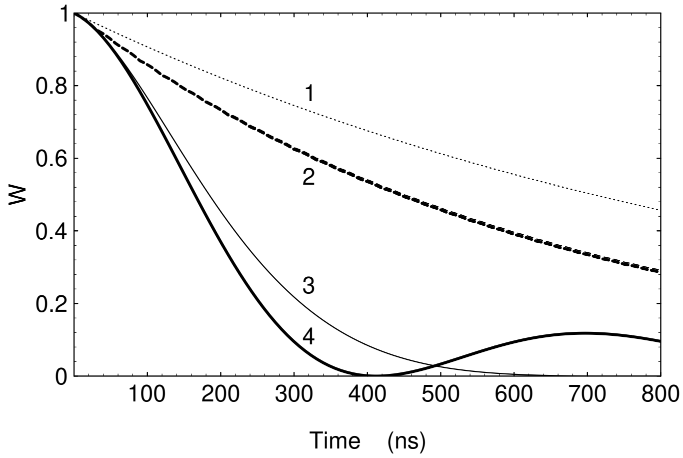

Fig. 4. Evolution of excited-state population W in two-level atom coupled to cavity mode with Lorentzian lineshape on resonance ($\Delta = 0$): curve 1—decay to background-mode continuum at rate $\gamma_b \simeq \gamma_f = 10^6$ s^{-1}; curve 3—uninterrupted decay in cavity with $F \equiv (1-R)^{-2} = 10^5$, L=15 cm, and f=0.02 ($\Gamma_s = 6.3 \times 10^6$ s^{-1}, $g_s = 4.5 \times 10^6$ s^{-1}); curve 4—idem, but with $F = 10^6$ ($\Gamma_s = 2 \times 10^6$ s^{-1}; damped Rabi oscillations); curve 2—interrupted evolution along *both* curves 3 and 4, at intervals $\tau = 3 \times 10^{-8}$ s.

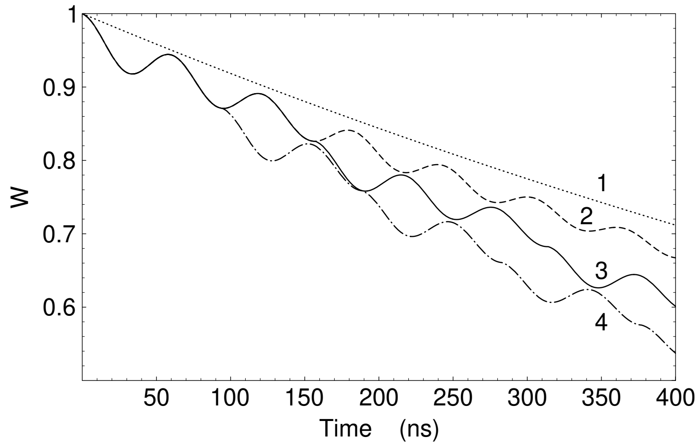

Fig. 5. Idem, for detuning $\Delta = 10^8 s^{-1}$ and $F = 10^6$: curve 1—decay to background-mode continuum; curve 2—uninterrupted free evolution; curve 3—interrupted evolution at intervals $\tau = 5\pi \times 10^{-8}$ s ($\Delta\tau = 5\pi$); curve 4—idem, for $\tau = 3\pi \times 10^{-8}$ s ($\Delta\tau = 3\pi$).

The resulting $|e\rangle \to |g\rangle$ fluorescence rate is collected and monitored as a function of the pulse repetition rate $1/\tau$. Each short, intense pump pulse of duration t_p and Rabi frequency Ω_p is followed by spontaneous decay from $|u\rangle$ back to $|e\rangle$, at a rate γ_u, so as to *destroy the coherence* of the system evolution, on the one hand, and *reshuffle the entire population* from $|e\rangle$ to $|u\rangle$ and back, on the other hand (Fig. 3). The demand that the interval between measurements significantly exceed the measurement time, yields the inequality $\tau \gg t_p$. The above inequality can be reduced to the requirement $\tau \gg \gamma_u^{-1}$ if the "measurements" are performed with π pulses: $\Omega_p t_p = \pi,\ t_p \ll \gamma_u^{-1}$. This calls for choosing a $|u\rangle \to |e\rangle$ transition with a much shorter radiative lifetime than that of $|e\rangle \to |g\rangle$.

Figure 4, describing the QZE for a Lorentz line on resonance ($\Delta = 0$), has been programmed for feasible cavity parameters: $\Gamma_s = (1 - R)c/L,\ g_s = \sqrt{cf\gamma_f/(2L)},\ \gamma_b = (1 - f)\gamma_f$, where R is the geometric-mean reflectivity of the two mirrors, f is the fractional solid angle (normalized to 4π) subtended by the confocal cavity, and L is the cavity length. It shows, that the population of $|e\rangle$ decays nearly-exponentially well within interruption intervals τ, but when those intervals become too short, there is significant inhibition of the decay. Figure 5 shows the effect of the detuning $\Delta = \omega_a - \omega_s$ on the decay: The decay now becomes oscillatory. The interruptions now *enhance* the decay, the degree of enhancement depends on the phase between interruptions.

3.2 Open-space reservoirs

The spectral response for hydrogenic-atom radiative decay via the $\vec{p} \cdot \vec{A}$ free-space interaction is given by [15]

$$G(\omega) = \frac{\alpha\omega}{[1 + (\omega/\omega_c)^2]^4}, \tag{15}$$

where α is the effective field-atom coupling constant and the cutoff frequency is

$$\omega_{\mathrm{c}} \approx 10^{19}\ \mathrm{s}^{-1} \sim \frac{c}{a_{\mathrm{B}}}. \tag{16}$$

Using measurement control that produces Lorentzian broadening [Eq. (7)] we then obtain

$$\kappa = \frac{\alpha\omega_{\mathrm{c}}}{3}\mathrm{Re}\left[\frac{f(2f^4 - 7f^2 + 11)}{2(f^2 - 1)^3} - \frac{6f\ln f}{(f^2 - 1)^4} - \frac{3i\pi(f^2 + 4f + 5)}{16(f + 1)^4}\right], \tag{17}$$

where

$$f = \frac{\nu - i\omega_a}{\omega_{\mathrm{c}}}. \tag{18}$$

In the range

$$\nu \ll \omega_{\mathrm{c}} \tag{19}$$

we obtain from Eq. (17) the *anti-Zeno effect* of accelerated decay. This comes about due to the *rising* of the spectral response $G(\omega) \approx \alpha\omega$ as a function of frequency (for $\omega \ll \omega_{\mathrm{c}}$). The Zeno effect can hypothetically occur only for $\nu \gtrsim \omega_{\mathrm{c}} \sim 10^{19}\ \mathrm{s}^{-1}$. But this range is well beyond the limit of validity of the present analysis, since $\Delta E \sim \hbar\nu \gtrsim \hbar\omega_{\mathrm{c}}$ may then induce other decay channels ("destruction") of $|e\rangle$, in addition to spontaneous transitions to $|g\rangle$.

4 Conclusions

Our unified analysis of two-level system coupling to field reservoirs has revealed the general optimal conditions for observing the QZE in various structures (cavities, waveguides, phonon reservoirs, and photonic band structures) as opposed to open space. We note that the wave-function collapse notion is not involved here, since the measurement is explicitly described as an act of dephasing (coherence-breaking). This analysis also clarifies that QZE cannot combat the open-space decay. Rather, impulsive or continuous dephasing are much more likely to accelerate decay by the inverse (anti-) Zeno effect.

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