ENTANGLEMENT SWAPPING WITH PDC SOURCES*

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We show that the possibility of distinguishing between single and two photon detection events is not a necessary requirement for the proof that recent operational realization of entanglement swapping cannot find a local realistic description. We propose a simple modification of the experiment, which gives a richer set of interesting phenomena.

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1 Introduction

The Bell theorem is a very important statement on the essential features of quantum mechanics. It has changed the nature of the earlier long disputes on the roots of probabilistic nature of quantum predictions. The answers to the basic questions are now expected to be found in laboratories, rather than during seminars on the philosophy of science. The performed experimental tests give a strong support for anyone expecting violations of Bell's inequalities also in (future) high collection efficiency experiments.

In 1989 it was shown [1] that the premises of the Einstein-Podolsky-Rosen argument for their claim that quantum mechanics is an incomplete theory are inconsistent when applied to entangled systems of three or more particles. First observations of the characteristic Greenberger-Horne-Zeilinger (GHZ) correlations were reported by Zeilinger and his group few months ago [2].

The technique to obtain GHZ correlations rests upon an observation that when a single particle from two independent entangled pairs is detected in a manner such that it is impossible to determine from which pair the single came, the remaining three particles become entangled. This method could only be developed with clear operational understanding of the necessary requirements to observe multi particle interference effects for particles coming from independent sources.

Until recent years it was commonly believed that particles producing EPR-Bell phenomena have to originate from a single source, or at least have to interact with each other. However, under very special conditions, by a suitable monitoring procedure of the emissions of the independent sources one can *pre-select* an ensemble of pairs of particles, which either reveal

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EPR-Bell correlations, or are in an entangled state. The first explicit proposal to use two independent sources of particles in a Bell test was given by Yurke and Stoler [3]. However, they did not discuss the importance of very specific operational requirements necessary to implement such schemes in real experiments. Such conditions were studied in [4] and [5].

The method of entangling independently radiated photons, which share no common past, [4] is essentially a pre-selection procedure. The selected registration acts of the idler photons define the ensemble which contains entangled signal photons (see next sections). Surprisingly, such a procedure enables one to realize the Bell's idea of "event-ready" detection. This approach for many years was thought to be completely infeasible and thus no research was being done in that direction [6]. This so-called *entanglement swapping* technique [4], was also adopted to observe experimental quantum states teleportation [7].

The first entanglement swapping experiment was performed in 1998 [8], [9]. High visibility (around 65%) two particle interference fringes were observed on a pre-selected subset of photons that never interacted. This is very close to the usual threshold visibility of two particle fringes to violate some Bell inequalities, which is 70.7%. Therefore there exists a strong temptation for breaking this limit, and in this way showing that the two particle fringes due to entanglement swapping have no local and realistic model.

However, due to the spontaneous nature of the sources involved, the initial condition for entanglement swapping cannot be prepared. Simply the probability that the two sources would produce a pair of entangled states each is of the same order as the probability that one of them produces two entangled pairs. In the latter case no entanglement swapping results. Nevertheless, such events can excite the trigger detectors (which in the case of the right initial condition select the antisymmetric Bell state of the two independent idlers). Therefore they are an unavoidable feature of the experiment, and have to be taken into account in any analysis of the possibility of finding a local realistic description for the experiment.

The aim of this paper is to perform such an analysis. We shall show that if all firings of the trigger detectors are accepted as pre-selecting the events for a Bell-type test, one must necessarily, at least partially, be able to distinguish between two and single photon events at the detectors observing the signals to enable demonstrations of violations of local realism. Whereas, if one accepts additional selection at the trigger detectors, based on the polarization of the idlers, detectors possessing this ability are unnecessary. We shall present our argumentation assuming that reader knows the methods and results of [4], [5] and [8]. The analysis will be confined to the gedanken situation of perfect detection efficiency (the results can be easily generalized to the non-ideal case).

2 Entanglement swapping and local realism

Consider the set-up of Fig. 1, which is in principle the scheme used in the Innsbruck experiment [8]. Two pulsed type-II down conversion sources are emitting their radiation into the spatial propagation modes a and b (signals), c and d (idlers). Due to the statistical properties of the PDC radiation, the initial state that is fed to the interferometric set-up has the following form:

$$|\psi\rangle = \sum_{n=0}^{\infty} \left[\frac{\gamma}{\sqrt{2}}(a_H^\dagger d_V^\dagger + a_V^\dagger d_H^\dagger)\right]^n \times \sum_{m=0}^{\infty} \left[\frac{\gamma}{\sqrt{2}}(e_H^\dagger b_V^\dagger + e_V^\dagger b_H^\dagger)\right]^m |0\rangle, \tag{1}$$

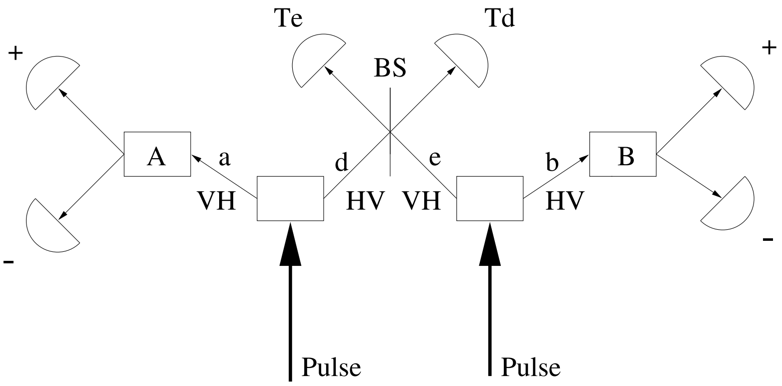

Fig. 1. Entanglement swapping. Two type-II down conversion crystals are pumped by a pulsed laser. The radiation from each of the crystals is entangled in polarizations (e.g. if one has an H polarized photon in mode a, then in mode b is a V polarized photon). The idler photons (in modes d and e) are fed into a non-polarizing beamsplitter BS. Simultaneous firing of the trigger detectors T_e and T_d pre-selects, in the event-ready way, a sub-ensemble of detection events behind the polarizing beamsplitters A and B. The orientation of the polarizing beamsplitters can be set at will by the local observers. The output signal photons are registered by the two local detection stations, consisting of detectors denoted by $+$ and $-$.

where, for instance, $a_H^\dagger$ denotes the creation operator of the photon in beam a having "horizontal" polarization. As for the entanglement swapping to work one cannot have too excessive pump powers [10], the γ coefficient can be assumed small. Therefore we select only those terms that are proportional to γ^2, as these are the lowest order terms terms that can induce simultaneous firing of both trigger detectors. They read

$$\begin{aligned} |\psi'\rangle &= \frac{1}{2}\gamma^2[(a_H^\dagger d_V^\dagger + d_V^\dagger d_H^\dagger)(e_H^\dagger b_V^\dagger + e_V^\dagger b_H^\dagger) \\ &+ (a_H^\dagger d_V^\dagger + a_V^\dagger d_H^\dagger)^2 + (e_H^\dagger b_V^\dagger + e_V^\dagger b_H^\dagger)^2]|0\rangle. \end{aligned} \tag{2}$$

The factor $\frac{1}{2}\gamma^2$ simply gives the order of magnitude of the probability of the two trigger detectors to fire, and therefore we drop it from further considerations. The action of the non-polarizing beamsplitter (BS) is described by $d_x^\dagger = \frac{1}{\sqrt{2}}(\tilde{d}_x^\dagger + i\tilde{e}_x^\dagger)$ and $e_x^\dagger = \frac{1}{\sqrt{2}}(\tilde{e}_x^\dagger + i\tilde{d}_x^\dagger)$ where $x = H$ or $x = V$, and $\tilde{e}$ and $\tilde{d}$ represent the modes monitored by the trigger detectors behind the beamsplitter. Taking into account only the terms in (2) that lead to clicks at two trigger detectors we arrive at

$$\begin{aligned} |\psi''\rangle = \Big[& i(a_H^{\dagger 2} + b_H^{\dagger 2})\tilde{e}_V^\dagger \tilde{d}_V^\dagger + i(a_V^{\dagger 2} + b_V^{\dagger 2})\tilde{e}_H^\dagger \tilde{d}_H^\dagger \qquad (3) \\ &+ i(a_H^\dagger a_V^\dagger + b_H^\dagger b_V^\dagger)(\tilde{e}_V^\dagger \tilde{d}_H^\dagger + \tilde{e}_H^\dagger \tilde{d}_V^\dagger) + \frac{1}{2}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)(\tilde{e}_V^\dagger \tilde{d}_H^\dagger - \tilde{e}_H^\dagger \tilde{d}_V^\dagger)\Big] |0\rangle. \end{aligned}$$

It is convenient to normalize and rewrite the above state into the form:

$$|\psi_N\rangle = \frac{1}{\sqrt{13}}\left[i\sqrt{2}\left(\frac{1}{\sqrt{2}}a_H^{\dagger 2} + \frac{1}{\sqrt{2}}b_H^{\dagger 2}\right)|VV\rangle + i\sqrt{2}\left(\frac{1}{\sqrt{2}}a_V^{\dagger 2} + \frac{1}{\sqrt{2}}b_V^{\dagger 2}\right)|HH\rangle\right.$$

$$
\begin{aligned}
&+ \quad \left(i(a_H^\dagger a_V^\dagger + b_H^\dagger b_V^\dagger) + \frac{1}{2}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)\right)|VH\rangle \\
&+ \quad \left(i(a_H^\dagger a_V^\dagger + b_H^\dagger b_V^\dagger) - \frac{1}{2}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)\right)|HV\rangle\bigg], \qquad (4)
\end{aligned}
$$

where $|VV\rangle = \tilde{e}_V^\dagger \tilde{d}_V^\dagger|0\rangle$, $|HH\rangle = \tilde{e}_H^\dagger \tilde{d}_H^\dagger|0\rangle$, $|VH\rangle = \tilde{e}_V^\dagger \tilde{d}_H^\dagger|0\rangle$ and $|HV\rangle = \tilde{e}_H^\dagger \tilde{d}_V^\dagger|0\rangle$. We see clearly that several processes may lead to the simultaneous firing of the trigger detectors (which observe the spatial modes) $\tilde{e}$ and $\tilde{d}$. The signal photons enter the polarizing beamsplitters. Their action can be described by the following relations

$$
x_V^\dagger = \cos(\theta_i)x_+^\dagger + \sin(\theta_i)x_-^\dagger, \qquad x_H^\dagger = -\sin(\theta_i)x_+^\dagger + \cos(\theta_i)x_-^\dagger, \qquad (5)
$$

with $x = a, b; i = 1, 2$ respectively and $+, -$ denoting the output spatial modes.

The probabilities of various two-particle processes that may at the spatially separated observation stations, under the condition of both trigger detectors firing simultaneously, are given by:

$$
\begin{aligned}
P(1a_+, 1a_-; 0b_+, 0b_-) &= P(2a_+, 0a_-; 0b_+, 0b_-) = P(0a_+, 2a_-; 0b_+, 0b_-) = \frac{2}{13}, \\
P(0a_+, 0a_-; 1b_+, 1b_-) &= P(0a_+, 0a_-; 2b_+, 0b_-) = P(0a_+, 0a_-; 0b_+, 2b_-) = \frac{2}{13}, \\
P(1a_+, 0a_-; 1b_+, 0b_-) &= P(0a_+, 1a_-; 0b_+, 1b_-) = \frac{1}{26}\sin(\theta_1 - \theta_2)^2, \\
P(1a_+, 0a_-; 0b_+, 1b_-) &= P(0a_+, 1a_-; 1b_+, 0b_-) = \frac{1}{26}\cos(\theta_1 - \theta_2)^2, \qquad (6)
\end{aligned}
$$

where, for example, $P(0a_+, 0a_-; 2b_+, 0b_-)$ denotes the probability of observing two photons at the output b_+, and no photons in the other outputs.

The Bell correlation function for the product of the measurement results on the signals at the two sides of the experiment can be redefined in the way proposed in [11]. All standard Bell-type events are assigned their usual values. For example, if there is a single photon click at the detector $\pm$ at the station A, the assigned value is ± 1. However, all non-standard events are assigned the value of one. I.e., if no photons are registered at one side, the local value of the measurement is one, if two photons are registered at one side again the local measurement value is one. The latter case includes both the event in which the two photons end-up at a single detector, as well as those when two detectors at the local station fire. Please note, that the experiment considered is a realization of Bell's idea of "event ready detectors" (see e.g. [6]). Therefore, non-detection events are operationally well defined (as the simultaneous firing of the trigger detectors pre-selects the sub-ensemble of time intervals in which one can expect the signal detectors to fire).

The above value assignment method works perfectly if one assumes that it is possible to distinguish between single and double photon detection at a single detector. However, this is usually not the case. Thus it is convenient to have a parameter α that measures the distinguishability of the double and single detection at one detector ($0 \leq \alpha \leq 1$, and gives the probability of distinguishing, by the employed devices, of the double counts).

The partial distinguishability blurs the distinction between events (at one side) in which there was a one photon detected at say the output $\pm$, and events in which two photons entered a the detector observing output $\pm$, but the detector failed to distinguish this event from a single photon count. In such a case the local event is sometimes ascribed by the local observer a wrong value namely -1 instead of 1 (if both photons go to the " $-$ " exit of the polarizer and the devices fail to inform the experimenter that it is a two photon event, this is interpreted as a firing due to a single photon and is ascribed a -1 value). Please note, that if one includes less than perfect detection efficiency of the detectors this problem is more frequent and more involved (we shall not study this aspect here).

Under such a value assignment the correlation function reads:

$$E_\alpha(\theta_1, \theta_2) = -\tfrac{1}{13}\cos(2\theta_1 - 2\theta_2) + \tfrac{4}{13}(1 + 2\alpha), \tag{7}$$

where α is the numerical value of the distinguishability. When we put into the standard CHSH inequality this correlation function it violates the standard bound of 2, only if the distinguishability satisfies $\alpha \geq \frac{9-\sqrt{2}}{8} \approx 0.948$. Such values are definitely beyond the current technological limits. As the efficiency of real detectors makes this problem even more acute, one has to propose a modification of the experiment that gets rid of this problem.

Therefore, in front of the idler detector Te we propose to put polarizing beamsplitter that transmit only vertical polarization whereas in front of the idler detector Td one that transmits only horizontal polarization. This further reduces the relevant terms in our state, i.e. those that can induce firing of the trigger detectors, to the following ones:

$$|\psi\rangle = \sqrt{\tfrac{2}{5}}\left(i(a_H^\dagger a_V^\dagger + b_V^\dagger b_H^\dagger) + \tfrac{1}{2}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)\right)\tilde{e}_V^\dagger \tilde{d}_H^\dagger |0\rangle. \tag{8}$$

Again we have normalized the above state.

Using the above formula we can calculate the probabilities of all possible processes in this interferometric set-up, conditional on firings of the two trigger detectors:

$$\begin{aligned}
P(1a_+, 1a_-; 0b_+, 0b_-) &= \frac{2}{5}\cos(2\theta_1)^2, \\
P(2a_+, 0a_-; 0b_+, 0b_-) &= P(0a_+, 2a_-; 0b_+, 0b_-) = \frac{1}{5}\sin(2\theta_1)^2, \\
P(0a_+, 0a_-; 1b_+, 1b_-) &= \frac{2}{5}\cos(2\theta_2)^2, \\
P(0a_+, 0a_-; 2b_+, 0b_-) &= P(0a_+, 0a_-; 0b_+, 2b_-) = \frac{1}{5}\sin(2\theta_2)^2, \\
P(1a_+, 0a_-; 1b_+, 0b_-) &= P(0a_+, 1a_-; 0b_+, 1b_-) = \frac{1}{10}\sin(\theta_1 - \theta_2)^2, \\
P(1a_+, 0a_-; 0b_+, 1b_-) &= P(0a_+, 1a_-; 1b_+, 0b_-) = \frac{1}{10}\cos(\theta_1 - \theta_2)^2.
\end{aligned} \tag{9}$$

Under the earlier defined value assignment the correlation function for the current version of the experiment reads:

$$E_\alpha(\theta_1, \theta_2) = -\frac{1}{5}\cos(2\theta_1 - 2\theta_2) + \frac{2}{5}(1 - \alpha)((\cos 2\theta_1)^2 + (\cos 2\theta_2)^2) + \frac{4}{5}\alpha. \tag{10}$$

When such a correlation functions are inserted into the CHSH inequality one has:

$$-2 \quad \leq \quad -\frac{1}{5}[\cos 2(\theta_1 - \theta_2) + \cos 2(\theta_1 - \theta_2') + \cos 2(\theta_1' - \theta_2) - \cos 2(\theta_1' - \theta_2')]$$

$$+ \quad \frac{4}{5}(1-\alpha)((\cos 2\theta_1)^2 + (\cos 2\theta_2)^2) + \frac{8}{5}\alpha \;\; \leq \; 2. \tag{11}$$

Please note that some of the terms of the correlation function which depend only on one local angle cancel upon insertion into CHSH inequality.

For $\alpha = 1$ (perfect distinguishability) the middle expression in (11) reaches 2.16569, i.e. we have a clear violation of the local realistic bound. This maximal violation occurs at angles (in radians) $2\theta_1 = -1.30278$, $2\theta_1' = -2.87435$, $2\theta_2 = 1.05326$, $2\theta_2' = 2.62386$. What is more interesting, for $\alpha = 0$, i.e. for a complete lack of distinguishability between two and single photon events at one detector, the expression in (11) reaches a value which is not much lower, namely 2.11453. This can be reached for the orientation angles $2\theta_1 = 0.0837317$, $2\theta_1' = -1.0749, 2\theta_2 = 3.05769, 2\theta_2' = 4.21568$).

Therefore we conclude that the proposed modification of the entanglement swapping experiment makes possible, despite the unwanted additional events due to the impossibility of controlling the spontaneous emissions at the two separate sources, makes it possible to consider it as test of Bell inequalities. The standard configuration can serve as a test of local realism only under the condition of extremely high distinguishability between two and single photon counts.

Finally let us mention that the proposed modification in the entanglement swapping configuration enables one to observe, in the event ready mode, a bosonic interference effect similar to the so-called Hong-Ou-Mandel dip [12]. It is described by the first four formulas of (9). For example, if $\theta_1 = \pi/4$, no coincidences between firings of the two detectors of the station a are allowed. All two photon events at this station are, under this setting, double counts at a single detector. Thus, we have two very interesting non-classical phenomena in one experiment.

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