acta physica slovaca vol. 49 No. 4, 627 – 632 August 1999

TWO–MODE CORRELATED STATES IN CAVITY WITH INJECTED ATOMS*

G. Ariunbold[ab1], **J. Peřina**[ac2], **Ts. Gantsog**[b3], **F.A.A. El–Orany**[ad4]
(a) *Department of Optics, Palacký University, 17. listopadu 50, 772 07 Olomouc, Czech Republic*
(b) *Department of Theoretical Physics of National University of Mongolia, 210646 Ulaanbaatar, Mongolia*
(c) *Joint Laboratory of Optics of Palacký University and Physical Institute of Academy of Sciences of Czech Republic, 17. listopadu 50, 772 07 Olomouc, Czech Republic*
(d) *Department of Mathematics and Computer Science, Faculty of Science, Suez Canal University, Ismailia, Egypt*

Received 26 April 1999, accepted 20 May 1999

We study a model of a lossless micromaser with two–level atoms interacting with a two-mode cavity field via two-photon transitions. We show that when the atoms are initially prepared in a superposition state then there is an operation regime of the micromaser when the cavity field evolves into a two-mode squeezed vacuum.

PACS: 42.50.Dv, 42.50.Gy

1 Introduction

In the past decade, there has been considerable interest in generating nonclassical states in a cavity with injected atoms. The one–atom maser or micromaser is a marvelous device which can answer some fundamental question in quantum mechanics and can be experimentally attained. The first one-atom maser realization has been reported by Meschede, Walther and Müller [1]. By using a lossless micromaser in which a quantized field is in a high–Q cavity with injected two–level atoms, there are many schemes that have been proposed to produce a number state [2–4] in the long interaction time regime, whereas, coherent [5–7] or displaced states [8] may be generated in the regime of a short interaction time and a large number of

*Presented at the 6th Central-European Workshop on Quantum Optics, Château Chudobín, Czech Republic, April 30 - May 3, 1999.

[1]E-mail address: ariunbold@optnw.upol.cz
[2]E-mail address: perina@risc.upol.cz
[3]E-mail address: gantsog@magicnet.mn
[4]E-mail address: el–orany@sloup.upol.cz

individual polarized atoms. A theory of the two–photon micromaser has been developed in some detail [9, 10] and experiments have been carried out [11]. The squeezed states may also be obtained in a lossless two–photon micromaser in the classical–like approximation [8, 12]. The central idea behind such arrangements is to reduce the atom–field interaction in the cavity to that in the one– or two–photon Jaynes–Cummings model. Instead, however, from theoretical point of view, one can consider the intensity–dependent interaction [13, 14] in a micromaser–like system [15, 16].

In the present work, we study the interaction of velocity–selected atomic beam in which atoms are prepared in a coherent superposition of the upper and lower states, and radiation through nondegenerate two–photon resonant transitions in a lossless cavity. Since the classical–like approximation enables us to obtain pure states as were studied earlier [8, 16], we show that two–mode transformed arbitrary states can also be generated in this micromaser context. Moreover, we rewrite the probability amplitude associated with ordinary two-mode squeezed states [17] into a slightly advanced form so that we analytically find its Gaussian quasidistributions [18, 19]. The next section is devoted to the derivation and the approximate solution of the recursion relation for the field density matrix elements, in comparison with certain numerical calculations.

2 Model and solution

By using the two–mode Jaynes–Cummings Hamiltonian [20], we can easily get for the field density–matrix elements the following recursion relation

$$\begin{aligned}
&\rho_N(n_1,n_2;n_1',n_2')\\
&=[\alpha^2 C_{n_1+1,n_2+1}C_{n_1'+1,n_2'+1}+\beta^2 C_{n_1,n_2}C_{n_1',n_2'}]\rho_{N-1}(n_1,n_2;n_1',n_2')\\
&+\beta^2 S_{n_1+1,n_2+1}S_{n_1'+1,n_2'+1}\rho_{N-1}(n_1+1,n_2+1;n_1'+1,n_2'+1)\\
&+\alpha^2 S_{n_1,n_2}S_{n_1',n_2'}\rho_{N-1}(n_1-1,n_2-1;n_1'-1,n_2'-1)\\
&+i\alpha\beta \mathrm{e}^{i\phi}C_{n_1+1,n_2+1}S_{n_1'+1,n_2'+1}\rho_{N-1}(n_1,n_2;n_1'+1,n_2'+1)\\
&+i\alpha\beta \mathrm{e}^{-i\phi}C_{n_1,n_2}S_{n_1',n_2'}\rho_{N-1}(n_1,n_2;n_1'-1,n_2'-1)\\
&-i\alpha\beta \mathrm{e}^{i\phi}S_{n_1,n_2}C_{n_1',n_2'}\rho_{N-1}(n_1-1,n_2-1;n_1',n_2')\\
&-i\alpha\beta \mathrm{e}^{-i\phi}S_{n_1+1,n_2+1}C_{n_1'+1,n_2'+1}\rho_{N-1}(n_1+1,n_2+1;n_1',n_2'), \qquad (1)
\end{aligned}$$

where $C_{n_1,n_2}=\cos(g\tau\sqrt{n_1 n_2})$, $S_{n_1,n_2}=\sin(g\tau\sqrt{n_1 n_2})$ and $\alpha,\ \beta,\ \phi$–being atomic parameters, $\alpha^2+\beta^2=1$. The solution of the recursion relation (1) can be found to be

$$\begin{aligned}
&\rho_N(n_1,n_2;n_1',n_2')\\
&\simeq \sum_{k=0}^{\downarrow n}\sum_{k'=0}^{\downarrow n'}\frac{(i\mathrm{e}^{-i\phi})^{k'-k}(\alpha\beta g\tau N)^{k+k'}\sqrt{n_1!n_2!n_1'!n_2'!}}{k!k'!\sqrt{(n_1-k)!(n_2-k)!(n_1'-k')!(n_2'-k')!}}\\
&\quad\times\rho_0(n_1-k,n_2-k;n_1'-k',n_2'-k'), \qquad (2)
\end{aligned}$$

under the conditions

$$\alpha,\beta\neq 0,\ N\gg 1,\ g\tau\sqrt{n_1 n_2}\ll 1.$$

The method adopted here was used in [8, 16]. In addition, the approximate analytical solution (2) must satisfy the weak condition as

$$|z| = |i\mathrm{e}^{\mathrm{i}\phi}\alpha\beta g\tau N| < 1, \tag{3}$$

since the normalization coefficient of the input field has not been changed after transformation.

3 Quasidistributions

As is seen from (2) if the system starts from a pure state $|\Psi_0\rangle$, then the cavity field evolves into the pure state, say $|\Psi\rangle$. For the sake of simplicity, we only concentrate on the special case, the input two–mode coherent state

$$|\Psi_0\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle \equiv |\alpha_1, \alpha_2\rangle, \tag{4}$$

where $|\alpha_i\rangle$ with $i = 1, 2$ defines an ordinary coherent state. Therefore, its transformation can be obtained from (2)

$$\langle n_1, n_2|\Psi\rangle \simeq (-z)^p \sqrt{\frac{p!}{q!}} \alpha_1^{n_1-p} \alpha_2^{n_2-p} L_p^{q-p}\left(\frac{\alpha_1\alpha_2}{z}\right), \tag{5}$$

here $p = \min(n_1, n_2), q = \max(n_1, n_2)$ and $L_p^{q-p}(x)$ are the Laguerre polynomials. Recently, Caves, Zhu, Milburn and Schleich have introduced a two–mode squeezed state [17]

$$\langle n_1, n_2|\tilde{\alpha}_1, \tilde{\alpha}_2\rangle_{(r,\eta)} =$$
$$\frac{(-\mathrm{e}^{i2\eta}\tanh r)^p}{\cosh r}\sqrt{\frac{p!}{q!}}\mu_1^{n_1-p}\mu_2^{n_2-p}L_p^{q-p}\left(\frac{\mu_1\mu_2}{\mathrm{e}^{i2\eta}\tanh r}\right)\mathrm{e}^{-(\tilde{\alpha}_1^*\mu_1+\tilde{\alpha}_2^*\mu_2)/2}, \tag{6}$$

here $\mu_{1,2} = \alpha_{1,2}/\cosh r$ and r being a squeeze parameter. The complex amplitudes $\tilde{\alpha}_{1,2}$ and $\alpha_{1,2}$ are given by [21]

$$\tilde{\alpha}_1 = \alpha_1 \cosh r - \mathrm{e}^{i2\eta}\alpha_2^* \sinh r,$$
$$\tilde{\alpha}_2 = -\mathrm{e}^{i2\eta}\alpha_1^* \sinh r + \alpha_2 \cosh r, \tag{7}$$

to establish the relation between squeezed, displaced or displaced, squeezed states. Its phase properties have been investigated by Gantsog and Tanaś [22] (for the initial vacuum state) and Peřinová, Lukš and Peřina [21]. In comparing expressions (5) and (6), it is clear that the transform amplitude z can be written as $z = \tanh r$ and when $r \to 0$, then $\cosh r \simeq 1$ and $\mu_{1,2} \simeq \alpha_{1,2}$; hence, the weak two–mode squeezed state can be generated in a micromaser.

Fortunately, the transformation leading to rather strong two–mode squeezing, can be obtained by direct numerical calculations of the recursion relation (1) for input four dimensional matrix elements. It is worth noting that because of atomic beam (pump) depletion, the squeezing amplitude $|z| = \tanh r$ can be approximated by the nonlinear function $\alpha\beta g\tau N - 0.265(\alpha\beta g\tau N)^2$ for $\alpha = \beta = 1/\sqrt{2}, \phi = 3\pi/2, g\tau = 0.001, N = 3000$ to get $r = 1.5$. The coefficient 0.265 is the same as that taken in [8, 16] and does depend upon the order of $g\tau$. Therefore, this nonlinearity makes it possible to call the approximation we have used

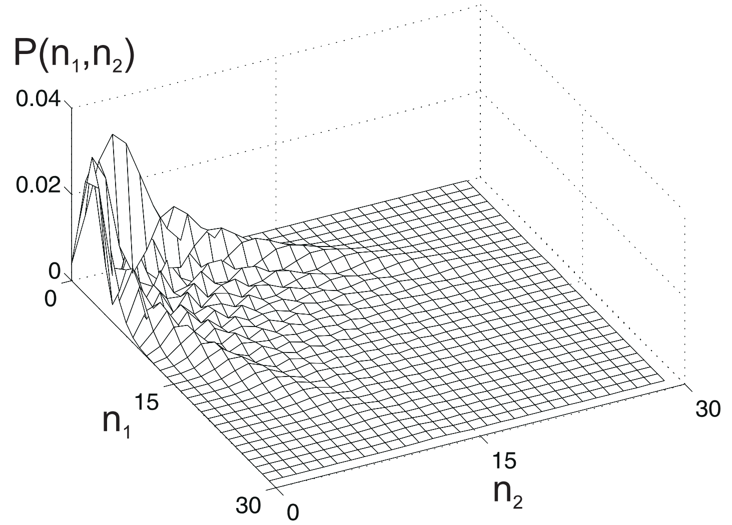

Fig. 1. Joint photon–number distribution $P(n_1, n_2) = \rho_N(n_1, n_2; n_1, n_2)$ of the generated field in the cavity as a function of mode numbers n_1 and n_2, with the parameters $\alpha = \beta = 1/\sqrt{2}, \phi = 3\pi/2, g\tau = 0.001$ and $N = 3000$ in the case of input two–mode coherent state $|\alpha_1, \alpha_2\rangle$ with $\alpha_1 = \alpha_2 = \tilde{\alpha}_1 \cosh r + \tilde{\alpha}_2^* \sinh r$ and $\tilde{\alpha}_1 = \tilde{\alpha}_2 = 1.0, r = 1.5$.

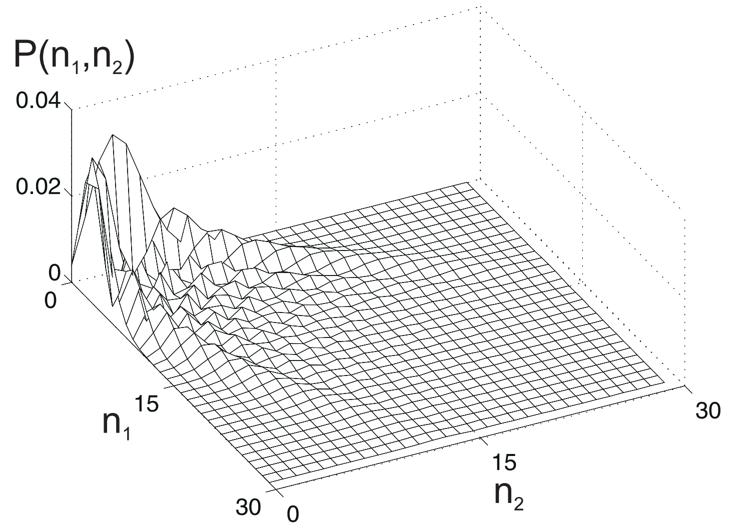

Fig. 2. Joint photon–number distribution for a two–mode squeezed state $|\tilde{\alpha}_1, \tilde{\alpha}_2\rangle_{(r,\eta)}$ discussed in [17], with $\tilde{\alpha}_1 = \tilde{\alpha}_2 = 1.0, r = 1.5, \eta = 0$.

as a classical–like instead of as a parametric (or classical), where $g\tau \propto 0, N \propto \infty$ and $g\tau N \propto$ constant. In Fig. 1, we have numerically demonstrated the joint photon–number distribution for a generated field for the input two–mode coherent state $|\alpha_1, \alpha_2\rangle$ with $\alpha_1 = \alpha_2 =$

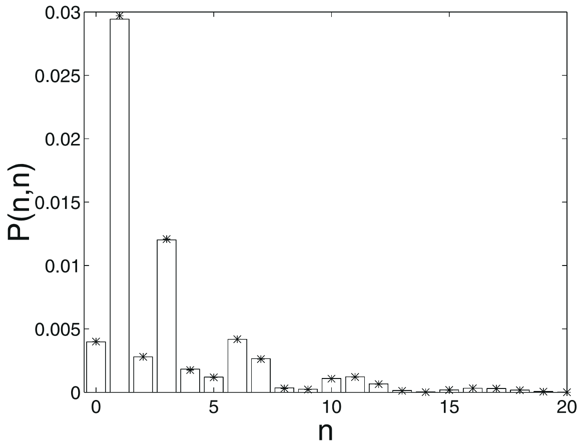

Fig. 3. Diagonal joint distributions $P(n,n)$ from Fig. 1 and Fig. 2 are plotted by "bars" and "stars", respectively.

$1.0\cosh 1.5 + 1.0\sinh 1.5$ using (1). Its comparison to the exact two–mode squeezed state (see, Fig. 2) is plotted in Fig. 3. As were mentioned in [8, 16], echo also exists in this micromaser context and the dependence of the field density matrix elements on the number of injected atoms indicates that due to the same initial atomic coherence (in general, with different but distinct phases) the emission of individual atoms is a co–operative process.

Moreover, with the help of the appropriate relation for the Laguerre polynomials adopted in [23, 24]

$$L_n^{m-n}(-x) = \frac{m!}{n!} x^{n-m} L_m^{n-m}(-x), \tag{8}$$

we can rewrite that probability amplitude for two–mode squeezed state (6) in the following reduced form

$$\langle n_1, n_2|\Psi\rangle = \frac{(-e^{i2\eta}\tanh r)^{n_2}}{\cosh r}\sqrt{\frac{n_2!}{n_1!}}\mu_1^{n_1-n_2} L_{n_2}^{n_1-n_2}\left(\frac{\mu_1\mu_2}{e^{i2\eta}\tanh r}\right) e^{-(\tilde{\alpha}_1^*\mu_1+\tilde{\alpha}_2^*\mu_2)/2}, \tag{9}$$

for any pair (n_1, n_2). Let us define Husimi Q-function [18]

$$Q(\beta_1, \beta_2) = \frac{1}{\pi^2}\langle\beta_1, \beta_2|\Psi\rangle\langle\Psi|\beta_1, \beta_2\rangle. \tag{10}$$

Now it is straightforward to obtain the quasidistribution by using new expression (9) and the generating function for the Laguerre polynomials, hence

$$Q(\beta_1, \beta_2) = \frac{1}{\pi^2\cosh^2 r} e^{-|\beta_1-\tilde{\alpha}_1|^2-|\beta_2-\tilde{\alpha}_2|^2-\tanh r(\beta_1^*-\tilde{\alpha}_1^*)(\beta_2^*-\tilde{\alpha}_2^*)-\tanh r(\beta_1-\tilde{\alpha}_1)(\beta_2-\tilde{\alpha}_2)}. \tag{11}$$

Here we put $\eta = 0$. It is clear that (11) is a special case of the more general Gaussian function which was considered by Peřina [18] and Agarwal [19]. After tracing over the second mode, the reduced Q–function becomes

$$Q(\beta_1) = \int Q(\beta_1, \beta_2) d^2\beta_2 = \frac{\mathrm{e}^{-|\beta_1 - \tilde{\alpha}_1|^2/\cosh^2 r}}{\pi \cosh^2 r}, \tag{12}$$

which is the Husimi function for the superposition of coherent amplitude $\tilde{\alpha}_1$ and quantum noise amplitude $\cosh r$.

Acknowledgments We thank Prof. V. Peřinová and Dr. A. Lukš for their stimulating discussions. This work was supported by Complex Grant No VS96028 of the Czech Ministry of Education.

References

[1] D. Meschede, H. Walther, G. Müller: *Phys. Rev. Lett.* **54** (1985) 551
[2] J. Filipowicz, J. Javanainen, P. Meystre: *J. Opt. Soc. Am. B* **3** (1986) 906
[3] J. Krause, M.O. Scully, H. Walther: *Phys. Rev. A* **36** (1987) 4547
[4] J. Krause, M.O. Scully, T. Walther, H. Walther: *Phys. Rev. A* **39** (1989) 1915
[5] J. Krause, M.O. Scully, H. Walther: *Phys. Rev. A* **34** (1986) 2032
[6] J.J Slosser, P. Meystre: *Phys. Rev. A* **41** (1990) 3867
[7] F.L. Kien, M.O. Scully, H. Walther: *Found. Phys.* **23** (1993) 177
[8] G. Ariunbold, J. Peřina, Ts. Gantsog: *J. Opt. B: Quant. Semiclass. Opt.* (1999) in press
[9] M. Brune, J.M. Raimond, S. Haroche: *Phys. Rev. A* **35** (1987) 154
[10] H. Moya-Cessa, P.L. Knight, A. Rosenhause-Dantsker: *Phys. Rev. A* **50** (1994) 1814
[11] M. Brune, J.M. Raimond, P. Goy, L. Davidovich, S. Haroche: *Phys. Rev. Lett.* **59** (1987) 1899
[12] G. Ariunbold, J. Peřina, Ts. Gantsog: *Acta Univ. Palacki Olomouc* **37** (1998) 29
[13] B. Buck, C.V. Sukumar: *Phys. Lett. A* **81** (1981) 132
[14] S. Singh: *Phys. Rev. A* **25** (1982) 3206
[15] I. Jex, H. Paul: *J. Mod. Opt.* **40** (1993) 2399
[16] G. Ariunbold, J. Peřina: *acta physica slovaca* **48** (1998) 315
[17] C.M. Caves, Ch. Zhu, G.J. Milburn, W. Schleich: *Phys. Rev. A* **43** (1991) 3854
[18] J. Peřina: *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, 2nd edn. (Kluwer Academic, Dordrecht, 1991)
[19] G.S. Agarwal: *Quant. Opt.* **2** (1991) 1
[20] S.-C. Gou: *Phys. Rev. A* **40** (1989) 5116
[21] V. Peřinová, A. Lukš, J. Peřina: *Phase in Optics* (World Scientific, Singapore, 1998)
[22] Ts. Gantsog, R. Tanaś: *Phys. Lett.* **152** (1991) 251
[23] A.M. Perelomov: *Generalized Coherent States and their Applications* (Springer–Verlag, Berlin, 1986)
[24] V. Peřinová, A. Lukš, J. Křepelka: *Phys. Rev. A* **53** (1996) 525