acta physica slovaca vol. 49 No. 4, 683 – 688 August 1999

ON THE BISTABILITY OF PARAMETRIC GENERATION PROCESS*

R. Filip[1]
Department of Optics, Palacký University,
17. listopadu 50, 772 07 Olomouc, Czech Republic

Received 30 April 1999, accepted 10 May 1999

Non-equilibrium steady state transitions in nonlinear parametric generation process are analyzed. When driven by external coherent light in signal and idler beams, the parametric generator exhibits a strongly bistable behaviour. Under certain circumstances, the bistabilities in signal and idler beams mutually compete. From presented analysis, it follows, the competition can in principle be controlled with the input light signals in a way to implement some model of measurement device. In particular, we suggest operation of the nonlinear parametric generator as an "all optical comparator", analogous to routinely used electronic devices.

PACS: 03.65.Bz, 42.65.S, 42.65.P, 42.50.-p

1 Introduction

Coherently driven nonlinear system can exhibit a variety of multistabilities. Under an action of weak external influence, the system may undergo a transition in the sense that one (or a set) of its macroscopic observables have substantial change. The finding of simple model of measurement device to illustrate significant quantum measurement phenomena is our motivation in investigation of these systems [3]. The parametrical models of nonlinear optics [1, 2] are simple and can be very suitable for this problem, but we need more discussions of the properties connected with application as the measurement device. In this paper, we will investigate classical equations of motion for coherently driven parametric generation process in up-conversion configuration inserted in an optical cavity [1, 4–6], and concern on the possibility of use of the light controllable competition of bistabilities in this system to construct the model of the all optical comparator as the element of classical measurement device. In the following paper we shall give a full quantum treatment of this system and analyze the properties of quantum measurement devices.

We consider the classical description of three modes $1, 2, 3$ oscillating with frequencies ω_1, ω_2, ω_3 $(\omega_1 = \omega_2 + \omega_3)$ in an optical cavity and coupled by the nonlinear parametric process [1].

* Presented at the 6th Central-European Workshop on Quantum Optics, Château Chudobín, Czech Republic, April 30 - May 3, 1999.

[1] E-mail address: filip@optnw.upol.cz

In addition, we assume that all the modes are damped by a proper loss-mechanism of cavity and modes 2, 3 are externally driven by the coherent pump signal with particular tuned frequencies ω_2 and ω_3 [5]. The mode 1 is assumed to be not stimulated at the input. Eliminating free oscillations, the dynamics of the system is described by the following classical equations for amplitudes of modes

$$\begin{aligned}\frac{d}{dt}A_1 &= -\gamma_1 A_1 + i\Gamma^* A_2 A_3, \quad \frac{d}{dt}A_2 = p_2 - \gamma_2 A_2 + i\Gamma A_3^* A_1,\\ \frac{d}{dt}A_3 &= p_3 - \gamma_3 A_3 + i\Gamma A_2^* A_1, \end{aligned} \tag{1}$$

where Γ are the complex nonlinear coupling constant, γ_i is the real damping constants of the mode, $p_2 = |p_2|\exp(i\psi_{P2})$, $p_3 = |p_3|\exp(i\psi_{P3})$ are the complex amplitudes of coherent pump.

Coherently driven nonlinear system above the threshold exhibits in long-time limit the strong coherent signal with small influence of classical fluctuation in all modes. Therefore, for investigation of the long-time behaviour, we use the linear correction method about the steady-state solution. We assume the solution in following form for all the modes $A_i(t) = \bar{A}_i + \delta_i(t)$, where $\delta_i(t)$ are corrections to solution of the steady-state equations

$$\gamma_1\bar{A}_1 = i\Gamma^*\bar{A}_2\bar{A}_3, \quad \gamma_2\bar{A}_2 = p_2 + i\Gamma\bar{A}_1\bar{A}_3^*, \quad \gamma_3\bar{A}_3 = p_3 + i\Gamma\bar{A}_1\bar{A}_2^*, \tag{2}$$

which satisfy the linearized equations

$$\begin{aligned}\frac{d}{dt}\delta_1 &= -\gamma_1\delta_1 + i\Gamma^*\bar{A}_3\delta_2 + i\Gamma^*\bar{A}_2\delta_3, \quad \frac{d}{dt}\delta_2 = -\gamma_2\delta_2 + i\Gamma\bar{A}_1\delta_3^* + i\Gamma\bar{A}_3^*\delta_1,\\ \frac{d}{dt}\delta_3 &= -\gamma_3\delta_3 + i\Gamma\bar{A}_1\delta_2^* + i\Gamma\bar{A}_2^*\delta_1. \end{aligned} \tag{3}$$

2 Steady-state analysis

For the symmetry between modes 2 and 3 we will discuss only the equations for one mode and the results for the other mode can be obtained by exchanging all parameters of modes 2 and 3.

Eliminating the variables $\bar{A}_1$ and $\bar{A}_2$ from (2), we obtain the following relations

$$\bar{A}_1 = \frac{i\Gamma^*}{\gamma_1}\frac{p_2}{\gamma_2 + \xi|\bar{A}_3|^2}\bar{A}_3, \;\; \bar{A}_2 = \frac{p_2}{\gamma_2 + \xi|\bar{A}_3|^2}, \;\; p_3 = \left(\gamma_3 + \frac{\xi|p_2|^2}{(\gamma_2 + \xi|\bar{A}_3|^2)^2}\right)\bar{A}_3, \tag{4}$$

where $\xi = \frac{|\Gamma|^2}{\gamma_1}$. There are two local extremes in $(|p_3|, |\bar{A}_3|)$ plane of the function $|p_3| = f_3(|\bar{A}_3|)$ in (4)

$$|\bar{A}_3|_{1,2} = \sqrt{\frac{2\sqrt{\eta}\cos(\frac{\pi\mp\phi}{3}) - \gamma_2}{\xi}}, \quad |p_3|_{1,2} = \left(\gamma_3 + \xi\frac{|p_2|^2}{4\eta\cos^2(\frac{\pi\mp\phi}{3})}\right)|\bar{A}_3|_{1,2}, \tag{5}$$

where $\eta = \xi\frac{|p_2|^2}{\gamma_3}$, $\cos\phi = \frac{2\gamma_2}{\sqrt{\eta}}$, and $\phi \in (0,\pi)$, if the condition $|\Gamma|^2|p_2|^2 > 4\gamma_1\gamma_2^2\gamma_3$ is satisfied. The extremal value $|\bar{A}_3|_1$ corresponds to the local minimum and the extremal value

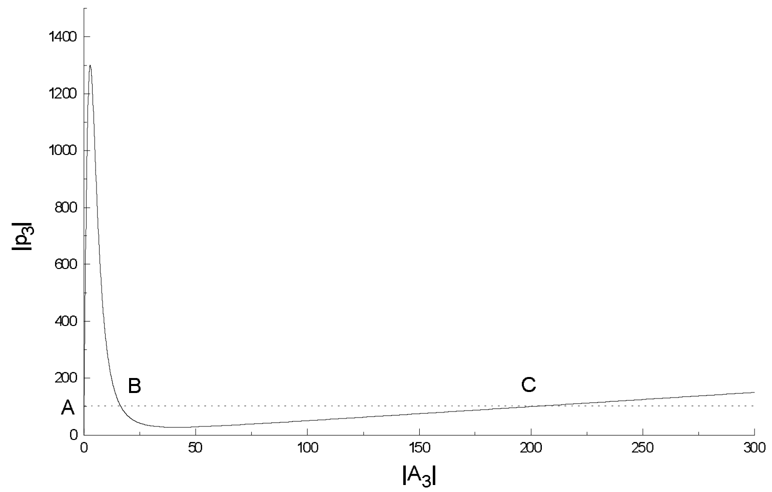

Fig. 1. The bistable behaviour in mode 3 for symmetric configuration, $p_2 = p_3 = 100$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.5$, $\Gamma = 0.1$ and $p_{2,3T} = 5$. The possible steady-state solutions are denoted by points A,B,C.

$|\bar{A}_3|_2$ corresponds to the local maximum as can be seen from the Fig. 1. There is a threshold for the existence of one more steady-state solution for mode 3, which is defined by the following conditions

$$|p_{2T}| = 2\frac{\sqrt{\gamma_1\gamma_2^2\gamma_3}}{|\Gamma|}, \qquad |p_3|_1 \leq |p_3| \leq |p_3|_2. \tag{6}$$

Analogically, by the elimination of variables $\bar{A}_1$ and $\bar{A}_3$ from the Eqs. (2), we can obtain the symmetrical relation for mode 2. In what follows we will assume that the system is sufficiently above both the thresholds for pump p_2, p_3 and the bistability occurs in both the modes.

If we try to find the considered steady-state solutions from the Eqs. (2), we obtain the fifth order polynomial equation for variable $|\bar{A}_3|$ and therefore we have no possibility of finding roots of this higher order polynomial by analytic methods and unfortunately the exact steady-state solutions of Eqs. (2) cannot be obtained analytically. Fortunately, for the system sufficiently above the threshold, the steady-state solution can be approximately predicted under the following approximations.

The steady-state solution A (see Fig. 1) for mode 3 can be approximately predicted under the assumption that $|\bar{A}_3|^2 \ll \frac{\gamma_1\gamma_2}{|\Gamma|^2}$ and $|p_2| \gg \frac{\sqrt{\gamma_1\gamma_2^2\gamma_3}}{|\Gamma|}$. Then from the Eqs. (4) we approximately obtain the steady-state solutions

$$\bar{A}_1 = i\frac{\gamma_2\Gamma^* p_2 p_3}{|\Gamma|^2|p_2|^2}, \quad \bar{A}_2 = \frac{p_2}{\gamma_2}, \quad \bar{A}_3 = \frac{\gamma_1\gamma_2^2 p_3}{|\Gamma|^2|p_2|^2}, \tag{7}$$

with conditions of validity $|p_3| \ll \frac{|\Gamma|}{\sqrt{\gamma_1\gamma_2^3}}|p_2|^2$ and $|p_2|^2 \gg \frac{\gamma_1\gamma_2^2\gamma_3}{|\Gamma|^2}$. The steady-state solution B (see Fig. 1) for mode 3 can be found under the approximation $\frac{\gamma_1\gamma_2}{|\Gamma|^2} \ll |\bar{A}_3|^2 \ll \sqrt{\frac{\gamma_1|p_2|^2}{\gamma_3|\Gamma|^2}}$ in

Eqs. (4),

$$\begin{aligned}\bar{A}_1 &= i\frac{\Gamma^*}{\gamma_1}\exp(i\psi_{P2}+i\psi_{P3})(\frac{\gamma_1^2|p_3||p_2|}{|\Gamma|^4})^{\frac{1}{3}},\\ \bar{A}_2 &= (\frac{\gamma_1|p_3|^2}{|\Gamma|^2|p_2|})^{\frac{1}{3}}\exp(i\psi_{P2}),\quad \bar{A}_3=(\frac{\gamma_1|p_2|^2}{|\Gamma|^2|p_3|})^{\frac{1}{3}}\exp(i\psi_{P3})),\end{aligned}\tag{8}$$

with condition of validity $\frac{|p_2|\sqrt{\gamma_1\gamma_3^3}}{|\Gamma|} \ll |p_3|^2 \ll \frac{|\Gamma|^2|p_2|^4}{\gamma_1\gamma_2^3}$. Finally, the steady-state solution C (see Fig. 1) for mode 3 is found by the approximation $\frac{\gamma_1\gamma_2}{|\Gamma|^2} \ll |\bar{A}_3|^2$ and $\sqrt{\frac{\gamma_1|p_2|^2}{\gamma_3|\Gamma|^2}} \ll |\bar{A}_3|^2$ in Eqs. (4) in following form

$$\bar{A}_1 = i\frac{\gamma_3\Gamma^* p_2 p_3}{|\Gamma|^2|p_3|^2},\quad \bar{A}_2=\frac{\gamma_1\gamma_3^2 p_2}{|\Gamma|^2|p_3|^2},\quad \bar{A}_3=\frac{p_3}{\gamma_3},\tag{9}$$

with condition of validity $|p_3|^2 \gg \frac{\gamma_1\gamma_2\gamma_3^2}{|\Gamma|^2}$ and $|p_3|^2 \gg \frac{\sqrt{\gamma_1\gamma_3^3}}{|\Gamma|}|p_2|$. The approximate steady-state solution for mode 2 can be obtained by exchanging parameters of modes 2 and 3.

The condition of linear asymptotical stability can be simply obtain from the Eqs. (3) in the following form

$$\frac{\gamma_1\gamma_2\gamma_3}{|\Gamma|^2} > 3\gamma_1|\bar{A}_1|^2 - \gamma_2|\bar{A}_2|^2 - \gamma_3|\bar{A}_3|^2.\tag{10}$$

From Eqs. (7), (8), (9) we can find that the steady-states A, C are stable, whereas the steady-state B is unstable, for the system sufficiently above the threshold.

3 Transient effect analysis

The description of transient effect is possible if the effect is sufficiently slow, for example, as can be seen in the Fig. 2. For fast transition effect, which grows of speed with increasing pump, we cannot distinguish a particular part of transition dynamics and the description is not possible.

The initial evolution of the system can be described by the short-time approximation in first order [1] of Eqs. (1). In this approximation, the interaction mixes the strong coherent pump with the initial signal in mode 2 and 3 particularly, analogous to the homodyne mixing. The small influence of random fluctuations in initial dynamics causes immediately random difference and the competition process can give a random result.

After initial mixing, slow dynamics of the system approximately achieves the unstable steady-state solution B, where the sensitivity on the fluctuations is maximal. Therefore, for investigation of fluctuations in the steady-state B we cannot use the linear correction theory, because the fluctuations can be very large and may no longer be treated as a small perturbation. The start of competition process, which occurs in this dynamical regime, can be described in the undepleted mode approximation of Eqs. (1). If we assume mode 1 in the steady-state B and states of modes 2, 3 are slightly different by initial signals $A_2(0)$ and $A_3(0)$ from the

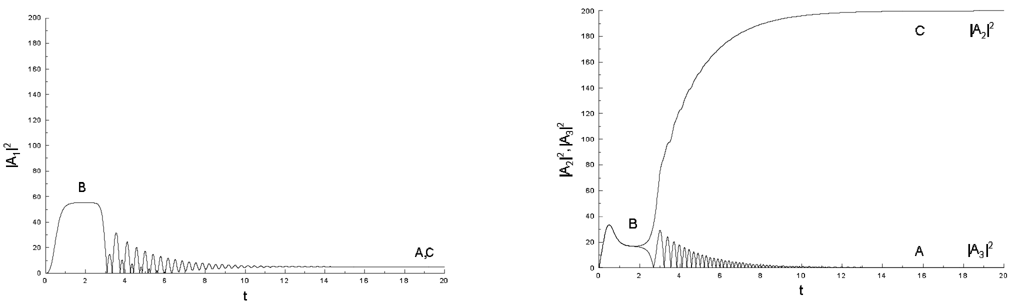

Fig. 2. The comparator effect in the solution of classical equation for simply symmetric configuration $p_2 = p_3 = 100$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.5$, $\Gamma = 0.1$, $A_1(0) = 0$, $A_2(0) = 0.02$, $A_3(0) = 0.01$ and $p_{2,3T} = 5$. The steady-state solutions A,B,C are denoted in correspondence with Fig. 1

steady-state B then, with the assumption $\psi_{P2} + \psi_{P3} = 2k\pi$, where $k = 0, 1, \ldots$, the following equations

$$\frac{d}{dt}A_2 = p_2 - \gamma_2 A_2 - |\Gamma \bar{A}_1| A_3^*, \;\; \frac{d}{dt}A_3 = p_3 - \gamma_3 A_3 - |\Gamma \bar{A}_1| A_2^*, \tag{11}$$

give to arise the competition process, if $|\Gamma \bar{A}_1|^2 > \gamma_2 \gamma_3$. This is the condition for unstable steady-state solution in the linearized model. Thus unstable behaviour is necessary for occurring competition effects. The equations are formally the same as the classical equations for the coherently driven frequency down-conversion process in the undepleted mode approximation [1, 2]. Because the mode 1 during competition behaviour is more and more depleted, the undepleted mode approximation has limited region of validity. The competition effect arising from down-conversion behaviour can lead to growth of one mode amplitude and depletetion of the other mode amplitude.

For explanation of dependence of competition effect on the initial signals, we start from the Eqs. (11) and find the equations for variables $W(t) = |A_2|^2 - |A_3|^2$ and $U(t) = \mathrm{Re}\{A_2\} - \mathrm{Re}\{A_3\}$, which are for symmetric case $p_2 = p_3 = p$ and $\gamma_2 = \gamma_3 = \gamma$ in the following form

$$\frac{d}{dt}W(t) + 2\gamma W(t) = pU(t), \qquad \frac{d}{dt}U(t) + (\gamma - |\Gamma \bar{A}_1|)U(t) = 0. \tag{12}$$

The condition $|\Gamma \bar{A}_1| > \gamma$ is sufficient for $|U(t)|$ uniformly increasing during start of competition. Then $|W(t)|$ increases and the competition gives rise, in principle, comparative behaviour, which exhibits strong dependence only on the sign of the initial difference of signals $U(0)$. The accuracy of symmetry conditions, which represent calibration of measurement device, is evidently important for the measurement accuracy.

If the competition effect is finished, the saturation effect occurs in winner mode. During the saturation process, the amplitude of mode 1 exhibits the relaxation oscillations to steady state solution and the slaving mode is damped more to negligible amplitude compared with the winner mode. Thus the saturation does not destroy the results of competition process. If the competition time is negligible compared to saturation time then the switching time can be estimated from driven damped oscillator behaviour in the order of $\tau \approx \frac{1}{\gamma}$.

In the final steady states, the pump is switching into the one of modes 2, 3 and weak signals are in the other modes. From steady-state solution we can see that the inequalities $|\bar{A}_2| \gg |\bar{A}_3|$ and $|\bar{A}_2| \gg |\bar{A}_1|$ are satisfied. Thus we can approximate the Eqs. (3) by more simple equations for classical fluctuations in the final steady state A

$$\begin{aligned}
\frac{d}{dt}\delta_1 &= -\gamma_1\delta_1 + i\Gamma^*\bar{A}_2\delta_3, \quad \frac{d}{dt}\delta_3 = -\gamma_3\delta_3 + i\Gamma\bar{A}_2^*\delta_1, \\
\frac{d}{dt}\delta_2 &= -\gamma_2\delta_2 + i\Gamma\bar{A}_1\delta_3^* + i\Gamma\bar{A}_3^*\delta_1, \qquad (13)
\end{aligned}$$

where the steady-state solutions are assumed in the form (7). The first and second equations are formally the same as the classical equations for the up-frequency conversion in undepleted mode approximation [1]. The third equation is completely driven by the dynamics of modes 1 and 3. Thus fluctuations in the final steady-state cannot be amplified in the linear noise approximation, which is in correlation with stable character of the final steady-state.

We can summarize that the classical comparative effect occurs in the system and can be simply described: for symmetrical configuration, the competition effect strongly depends only on the sign of initial difference of signals $U(0)$ and thus actually realize the comparison of the quadratures of these initial signals. Because the comparator is uniformly sensitive to a very small difference $U(0)$ between inputs, we can expect that is appropriated for the model of quantum measurement device.

Acknowledgments I would like to thank Richard Horák, Jan Peřina and Zdeněk Hradil and my colleagues for useful discussions. This research was supported by the grant No VS96028 of the Czech Ministry of Education and by the research project CEZ: 314/98 "Wave and particle optics" of the Czech Ministry of Education.

References

[1] J. Peřina: *Quantum Statistics of Linear and Nonlinear Optical Phenomena* (Kluwer Acadamic Publishers, Dordrecht, 1991)

[2] D.F. Walls, G.J. Milburn: *Quantum Optics* (Springer-Verlag, Berlin, 1994)

[3] D.F. Walls, M.J. Collet, G.J. Milburn: *Phys. Rev.* **D 32** (1985) 3208

[4] K. Germey, F.-J. Schütte, R. Tiebel: *Ann. Phys. (Leipzig)* **39** (1982) 170

[5] F.-J. Schütte, K. Germey, R. Tiebel, K. Worlitzer: *Optica Acta* **30** (1983) 465

[6] K. Germey, F.-J. Schütte, R. Tiebel: *Opt. Commun.* **69** (1989) 438