ELECTRON ANTIBUNCHING*

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Two-electron correlation function is introduced and the basic property of multiparticle electron correlations — antibunching — is derived from its form. Two-particle correlations of photons and electrons are compared as well as the influence of a Wien filter on one- and two-electron coherence.

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1 Two-electron correlation function

The two-electon correlation function expresses the probability density of finding an electron at the point r_1 at time t_1 and another electron at the point r_2 at time t_2. It can be calculated as follows:

$$G_{12}^{(2)} = \left\langle \hat{\psi}_1^\dagger \hat{\psi}_2^\dagger \hat{\psi}_2 \hat{\psi}_1 \right\rangle = \mathrm{Tr}\{\hat{\rho}\, \hat{\psi}_1^\dagger \hat{\psi}_2^\dagger \hat{\psi}_2 \hat{\psi}_1\}, \tag{1}$$

where $\hat{\rho}$, $\hat{\psi}_i$ and $\hat{\psi}_i^\dagger$ $(i = 1, 2)$ is the density operator of the electron field and field operators annihilating and creating an electron at the space-time point (r_i, t_i), respectively. Under particular circumstances that are usually fulfilled in practical experiments (the most important one is the quasimonochromaticity of the electrons), the probability of *detecting* two electrons at the points r_1, r_2 at times t_1, t_2 is proportional to the correlation function $G_{12}^{(2)}$.

The two-electron correlation function has been calculated for a fermion chaotic state that is believed to be produced by all the currently available electron sources as a thermal source, field emission source etc. (for description of the electron chaotic state see [2], for the calculation itself see [3]). The chaotic state was introduced by Glauber [1] for light as a state with a maximum entropy. It is a state in which the individual modes of the field are mutually uncorrelated.

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The density operator of such a state can be expressed as a direct product of one-mode density operators. These one-mode density operators obey the corresponding quantum statistics, i.e., the Bose-Einstein statistics in the case of bosons and the Fermi-Dirac statistics in the case of fermions. In the latter case, the one-mode density operators are very simple because there are only two possibilities of occupation of each mode of the field – either there is one electron or there is none. The correlation between different modes that might appear due to the Coulomb interaction between electrons on the way from the source to the detectors can be neglected because of large distances between electrons in the beam.

After a nontrivial calculation based on the fermion anticommutation relations, we arrive at the result

$$G_{12}^{(2)} = \begin{vmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{vmatrix} = \Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21}, \tag{2}$$

where $\Gamma_{ij} = \left\langle \hat{\psi}_i^\dagger \hat{\psi}_j \right\rangle$ is the cross-correlation function of the electron field at the points (r_i, t_i) and (r_j, t_j). Upon realizing that Γ_{11} and Γ_{22} are at the same time the one-electron correlation functions $G_1^{(1)}$, $G_2^{(1)}$ at the points (r_1, t_1) and (r_2, t_2), respectively, the *normalized* two electron correlation function can be expressed as

$$g_{12}^{(2)} \equiv \frac{G_{12}^{(2)}}{G_1^{(1)} G_2^{(1)}} = 1 - |\gamma_{12}|^2, \tag{3}$$

where

$$\gamma_{12} = \frac{\Gamma_{12}}{\sqrt{\Gamma_{11}\Gamma_{22}}} \tag{4}$$

is the complex degree of coherence of the electron field at the points (r_1, t_1) and (r_2, t_2). For stationary fields γ_{12} depends on the difference $\tau = t_2 - t_1$, so we can write $\gamma_{12} = \gamma(r_1, r_2, \tau)$. From Eq. (3) it follows that if $|\gamma_{12}| > 0$, i.e., if the field at the first detector at the time t_1 is not completely incoherent with the field at the second detector at the time t_2, the probability of detecting two electrons at two spacetime points is less than the product of the probabilities of the individual detection processes. If we put $r_1 = r_2$, which corresponds to the situation of detecting consequently two electrons at one detector, then $g_{12}^{(2)} \to 0$ as τ goes to zero because $|\gamma_{12}|$ then converges to unity. The detection of two electrons short after one another is thus very unlikely, which suggests that electrons avoid coming to a detector in pairs. This phenomenon is called *antibunching* and seems to be a fundamental property of all fermion beams as will be shown later.

2 Comparing the two-electron and two-photon correlations

If we compare the result (3) with the normalized two-photon correlation function for thermal light according to Mandel and Wolf [4]

$$g_{12}^{(2)} = 1 + |\gamma_{12}|^2, \tag{5}$$

the similarity of both the expressions is obvious. The fundamental difference, however, is the sign of the term $|\gamma_{12}|^2$. This sign arises from the symmetry and antisymmetry of bosonic and fermionic states, respectively, under permutation of particles.

If we put similarly as for electrons $r_1 = r_2$ for thermal photons, we see that $g^{(2)}_{1,2} \to 2$ if $t_2 \to t_1$. On the other hand, if the time difference $t_2 - t_1$ is large (exacly speaking, larger than the coherence time of the light), γ_{12} goes to zero and $g^{(2)}_{1,2} = 1$. It is therefore more likely to detect two photons short after one another than long after one another. It means that thermal photons prefer coming to a detector in pairs, which is called bunching. Bunching was first observed indirectly in 1956 by astronomers Hanbury Brown and Twiss [5] and directly in 1957 by Twiss, Little and Hanbury Brown [6].

In the following we will show that antibunching seems to be a fundamental property of fermion beams, which cannot be said about bunching in the case of photons. For this reason, we explore some properties of the two-electron correlation function for a general state, i.e., not just the chaotic one.

The following anticommutation relation holds for electron annihilation field operators $\hat{\psi}_1$ and $\hat{\psi}_2$:

$$\hat{\psi}_2\hat{\psi}_1 = -\hat{\psi}_1\hat{\psi}_2. \tag{6}$$

From this it follows immediately that if the two points (r_1, t_1) and (r_2, t_2) are identical, the operator product Eq.(6) becomes zero and the correlation function $G^{(2)}(r_1, t_1, r_2, t_2)$ vanishes. This is of course nothing else than the Pauli principle excluding the possibility of finding two fermions in the same state. In situations we meet in nature there are rarely discontinuities or singularities. Therefore it can be expected that if the points (r_1, t_1) and (r_2, t_2) are not identical but very close to each other in both the temporal and spatial sense, the correlation function $G^{(2)}_{12}$ will not differ very much from zero. In other words, it approaches zero continuously as the point (r_2, t_2) approaches (r_1, t_1). The detection of two electrons at one detector short after one another is thus even in this general situation very unlikely, which is precisely antibunching.

We must point out, however, that the situation is a bit more complicated. It might happen, for example, that the correlation function $g^{(2)}_{12}$ would be very small in some region around zero, then reach the value of, say, 3 and only after that go to unity. Then it is difficult to say globally if there is bunching or antibunching and we could observe even pure bunching in a coincidence experiment (see Sec. 3) – because of a bad time resolution it might happen that the parts of g above unity would surpress the parts under unity. Nevertheless, if antibunching is defined just by the properties of $g^{(2)}_{12}$ for $\tau \to 0$, there is always antibunching in the case of fermions.

The situation with light is quite different. There exist photon states that do not show bunching and even states for which *antibunching* is observed. Let us concentrate on the case of a photon field emitted from a one-mode laser. This field can be considered to be in a coherent state, which is an eigenstate of the operator that annihilates a photon in this mode. The detection of a photon therefore does not change the state of the field and it hence does not influence the probability of detecting another photon. The two-photon correlation function thus factorizes into a product of the two corresponding one-photon correlation functions which means that the detections at the two space-time points (r_1, t_1) and (r_2, t_2) are statistically independent. The normalized two-photon correlation function is equal to unity for all τ. For laser light we therefore do not observe bunching, photons do not tend to come into a detector in pairs.

Comparing the two-particle correlation properties of electron and photon field, we see that antibunching occurs for *all electron states*, while bunching does *not* occur for *all photon states*. From this it follows that the difference between electrons and photons is not only the sign in the correlation function or something similar and we also cannot simply apply everything what is known for photons on the case of electrons, only changing some signs from plus to minus. Another complication in seeking the analogy between electron and photon correlations is that there exist no coherent states and no P-representation for electrons, which makes the calculations much more complicated compared to quantum optics.

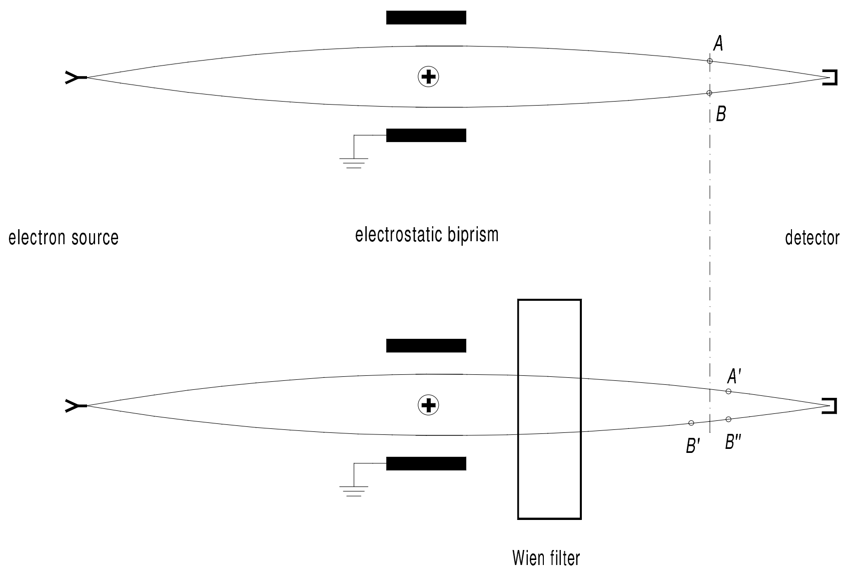

Fig. 1. Influence of the Wien filter on the one-electon coherence

3 Comparing one- and two-electron interference

It is a well-known fact from optical interferometric experiments that if there is a path difference between the two interfering beams larger than the coherence length of the light, the interference fringes vanish. Such a path difference can be introduced e.g. by inserting a medium with a different index of refraction into one interferometer arm. Vanishing of the fringes is usually explained by loss of longitudinal coherence between the two interfering beams.

For electrons an analogous path difference can be introduced by the Wien filter, i.e., by crossed homogeneous electrostatic and magnetostatic fields perpendicular to each other and to the path of the electrons. The same effect on the interference field as in the case of light is observed, namely, vanishing of fringes caused by loss of coherence between the two interfering beams [7]. This fact might suggest that if a correlation experiment with electrons is performed, and a longitudinal shift by the Wien filter is introduced, also antibunching should vanish as a result of the loss of coherence of the beams coming to the first and second detectors. This is not right, however, as we will show in the following.

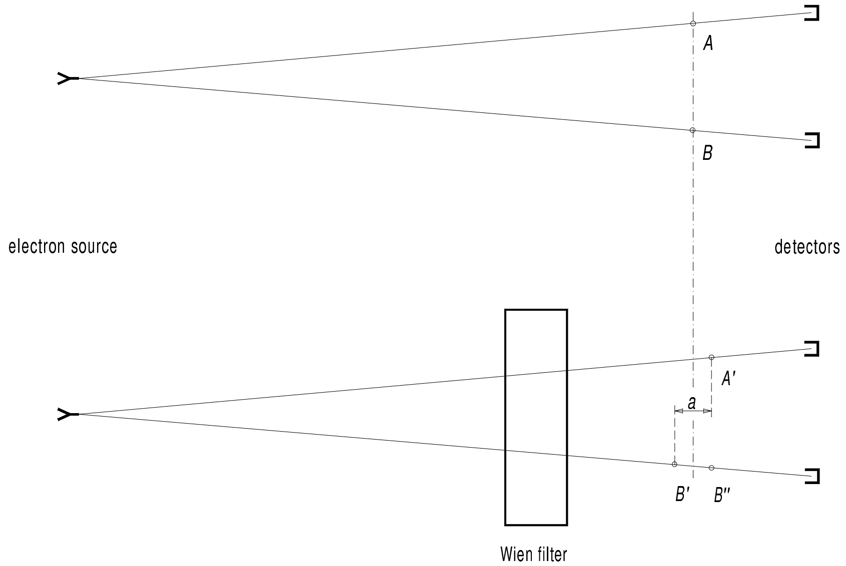

Fig. 2. Influence of the Wien filter on the one-electon coherence

Let us see in more detail what really happens in the interferometer if the Wien filter is used. It is of fundamental importance to realize that the coherence between the field coming to the detector through the left and the right arm of the interferometer is not lost, but only hidden[1].

Figure 1 shows this situation: without the Wien filter [Fig. 1(a)], the field at the point A is coherent with the field at the point B at the same moment of time. With the Wien filter [Fig. 1(b)], the field at the point A' is coherent with the field at the point B' but not B'' thanks to the longitudinal shift caused by the Wien filter. Therefore the partial fields coming through the left and right interferometer arms to the detector *simultaneously* are mutually incoherent and no inteference fringes can therefore be observed.

The situation in the two-electron correlation experiment with two detectors and a Wien filter is different (see Fig. 2). In fact, for quasimonochromatic electrons a longitudinal shift by a length a caused by the Wien filter is equivalent to a temporal shift by a time $t_a = a/v$, where v is the average velocity of the electrons. This suggests that the Wien filter will cause only a temporal shift of the correlation function $g_{12}^{(2)} = g^{(2)}(r_1, r_2, \tau)$ in the variable $\tau = t_2 - t_1$ by the time t_a. Really, in the experimental setup without the Wien filter [Fig. 2(a)] two electrons cannot be found simultaneously at the points A and B because the field at these points is mututally coherent and therefore they cannot come to the two detectors simultaneously (antibunching). With the Wien filter [Fig. 2(b)], two electrons cannot be found simultaneously at the points A' and B' (but can be found at A' and B''). Therefore no two electrons can come to the two detectors after one another with the delay equal to the time that an electron needs to pass the distance a. As this time is precisely t_a, the correlation function $g_{12}^{(2)}$ will be just shifted by t_a.

[1]this was shown even experimentally in 1993 by F. Hasselbach [7]

The longitudinal shifts a caused by the Wien filter are typically about a few coherence lengths. This corresponds to temporal shifts t_a of about a few coherence times. A typical coherence time of field-emission electrons is 10^{-14} s. From this it follows that even with the fastest available electronics enabling the time resolution of 10^{-11} s it is *not possible to observe the influence of the Wien filter on the two-electron correlations*. On the other hand, the *influence of the Wien filter on the one-electron correlation function* in an interferometer, i.e., on the intensity in the interference region, *is strong* and is observed almost daily in electron interference experiments.

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