acta physica slovaca vol. 49 No. 4, 707 – 712 August 1999

THREE-PHOTON STATES FOR QUANTUM TELEPORTATION*

M. Koniorczyk[1], J. Janszky, Z. Kis
Department of Nonlinear and Quantum Optics, Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, P.O. Box 49 H-1525 Budapest, Hungary

Received 28 April 1999, accepted 10 May 1999

A three-particle generalization of the quantum teleportation of polarization states is discussed. A possible nonlinear optical process is discussed, which can lead the required EPR-states. The Bell-state analysis of our three-photon Bell-states applying a beam-splitter and polarization analyzers is discussed.

PACS: 03.67.-a, 03.65.Bz, 42.50.Dv

1 Introduction

Parametric down-conversion is the most commonly used process to generate non-classical light today. The properties of multiparticle superpositions are the central issues of quantum mechanics. Second order nonlinear optical processes have made twoparticle entanglement experimentally available. This lead to the capability of studying Bell-inequalities experimentally, and recently to the realization of nonlocality's maybe most spectacular manifestation: quantum teleportation.

Three-particle interferometry may provide an even larger variety of nonclassical phenomena, including entanglement. Though experimental techniques of generating three-particle entangled states are unsettled at the present time, theoretical investigation of the possible phenomena is an interesting field.

In this paper we present a partially degenerate third-order parametric down–conversion process. In the light generated by this process, photon-pairs and photons are correlated with each other. We call this state PDGHZ (Partially Degenarte GHZ) states. It has been shown, that in certain circumstances superpositions of one- and two-photon states can be teleported with the aid of such entangled states. We discuss interferometric Bell-state analysis in this case.

*Presented at the 6th Central-European Workshop on Quantum Optics, Château Chudobín, Czech Republic, April 30 - May 3, 1999.
[1]E-mail address: kmatyas@bird.szfki.kfki.hu

2 The partially degenerate third order down-conversion process

Three-photon down-conversion processes produce novel quantum states of light. In a totally degenerate third order down-conversion process, where the photons of the pump decay into three photons in the same spatial mode, the so called "star states" are produced (named after the shape of their Wigner-function). These states possess interesting interference-properties [3], and their efficiency in three-photon absorption processes is known to be superior to to that of a coherent state with two orders of magnitude [4]. This process in a cavity has also been studied [5]. The nondegenerate process, in which all the generated photons are in separate spatial modes, yield the famous GHZ (Greenberger-Horne-Zeilinger) states [2], important in multiparticle interferometry [1].

Let us consider a three-photon down-conversion process, in which two photons are generated in the same spatial mode, and one photon in an other, separate mode.

Parametric approximation neglects the effect of quantum-fluctuations [9] and the entanglement of the field modes [10], which become very important in higher order processes. Another problem with parametric approximation for higher order processes is the photon-number divergence [6]. We examine the process with quantized pump, and numerical diagonalizaton of the interaction Hamiltonian. This method, same as the one applied in e.g. [7] and [8] reveals the most important features of the process under consideration.

The Hamiltonian of the system is

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{int} = \hbar\omega(3\hat{a}^+\hat{a} + \hat{b}^+\hat{b} + \hat{c}^+\hat{c}) + \hbar g(\hat{a}\hat{b}^{+2}\hat{c}^+ + \hat{a}^+\hat{b}^2\hat{c}), \tag{1}$$

where $\hat{a}$, $\hat{b}$ and $\hat{c}$ are the absorption operators of the pump, signal and idler modes, respectively. We shall call "signal" the mode with the two photons. An important feature of the Hamitonian (1) is, that the unperturbated part $\hat{\mathcal{H}}_0$ and the interaction Hamiltonian $\hat{\mathcal{H}}_{int}$ commute. Thus the total energy of both modes described by $\hat{\mathcal{H}}_0$ is conserved by the interaction $\hat{\mathcal{H}}_{int}$. This enables us to use the interaction picture by simply dropping $\exp(-it\hat{\mathcal{H}}_0/\hbar)$, so that the state of the field at time t is:

$$|\Psi(t)\rangle = \exp(-it\hat{\mathcal{H}}_{int}/\hbar)|\Psi(0)\rangle, \tag{2}$$

where $|\Psi(0)\rangle$ is the initial state of the filed. The Hamitonian (1) describes the PDGHZ process, and a special kind of third harmonic generation as well. The initial conditions distinguish between the cases. We assume the pump field initially to be in the $|\alpha\rangle$ coherent state, and vacuum in the signal and idler modes, that is

$$|\Psi(0)\rangle = \sum_{n=0}^{\infty} B_n|n, 0, 0\rangle, \tag{3}$$

where $|n, k, l\rangle$ refers to the number-state with n,k,l photons in the pump,signal and idler modes respectively, and $B_n = \exp(-|\alpha|^2)\frac{|\alpha|^n}{\sqrt{n!}}e^{i\, n\arg(\alpha)}$ the coefficients of the initial coherent state.

Exploiting, that the Hamitonian (1) has the $n+1$-dimensional invariant subspaces $\{|n-k, 2k, k\rangle\}_{k=0\ldots n}$ for each n, the state at time t can be written as:

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} B_n \sum_{k=0}^{n} C_{nk}(t)|n-k, 2k, k\rangle, \tag{4}$$

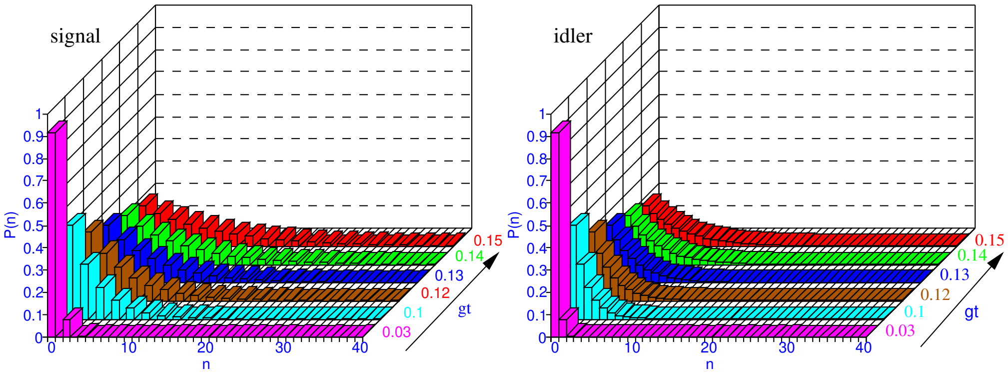

Fig. 1. Photon statistics of the signal and idler output modes of PDGHZ process in the parametric regime. The reduced density matrices of both modes is diagonal, but the two modes correlated perfectly. In the signal mode only photon-pairs are present.

and the $C_{nk}(t)$ coefficients can be calculated numerically.

The density matrices of the field modes in the modes can be determined by tracing out the other modes in the total density matrix defined by (4).

The interaction time scales with the g coupling constant in Eq. (1). The behavior of the system is similar to parametric process, when the output modes and the pump are approximately not entangled, and the energy flow from the signal and the idler to the pump is negligible. We found, that the above conditions are true only for small (≈ 0.1) values of scaled time. Because of the small g values in third-order processes, if the process could experimentally realized, probably this period of the evolution could describe the output field properly (though the long-time evolution, not discussed here is also interesting theoretically). We remark, that the output-pump entanglement is the main limit of parametric approximation in this case.

Tracing out the pump and signal (or the pump and idler) modes, the reduced density matrix becomes diagonal. This means, that the signal (idler) mode, if considered itself, is of classical character, in the sense that their phase is random. Thus they are described by the photon statistics in Fig. 1. The idler mode is a thermal noise, while in the signal mode, photon pairs appear. Indeed, the signal and idler modes are perfectly correlated, since any photon in the idler mode is generated together with a photon-pair in the signal mode. We are planning to study the details of this correlation in the future. In a weak process, because of the small gt value, mainly correlated photons photon pairs will appear in the output. This is similar to the second order parametric down-conversion.

We conclude, that using a PDGHZ process, creation of entangled photons and photon-pairs is feasible.

3 Three-photon Bell-states and quantum teleportation

Suppose, that the process described in Sec. 2 is available in a nonlinear optical crystal with "eooe" phase-matching. In this case, the situation in Fig. 2 can appear. Similarly to the case of generating EPR states with a BBO crystal [11] (the second-order case, used in the Inns-

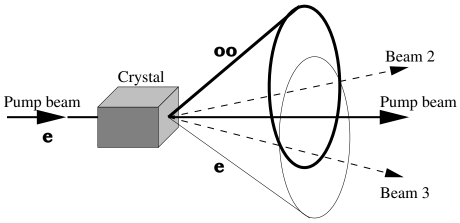

Fig. 2. Configuration for generating entangled one and two photon states using a PDGHZ process with proper phase-matching. The pump photons (thick line) decays to three photons, two of them going to the same direction (thicker cone), one to an other direction (thinner cone). The beams are polarized due to the phase-matching condition (the letters "e" and "o" in the figure correspond to ordinary and extraordinary beams. Selecting the intersection of the two cones (dashed line) with a proper diaphragm, these beams are in state $|\Psi^{(-)}\rangle$. This process – if found in some nonlinear optical medium –, would be useful for generating the entangled states required for particle number teleportation.

bruck teleportation experiment [12]) , in the intersection of the two light-cones, the following entangled state emerges:

$$|\Psi^{(-)}\rangle = \frac{1}{\sqrt{2}}(|\leftrightarrow_2\rangle|\Updownarrow_3\rangle - |\Updownarrow_2\rangle|\leftrightarrow_3\rangle). \quad (5)$$

In (5), the single and double stroked arrows refer to one and twophoton states, while the numbers 2 and 3 in the indices refer to the two spatial modes. This expresses, that in the two beams the polarization and the photon-number is uncertain, but the two modes are strictly correlated. This is the EPR state for our teleportation scheme.

Envisage now the apparatus in Fig. 3, with the EPR-source being the configuration described above. We shall use the notation of Fig. 3 for the spatial modes in the following. The state to be teleported is:

$$|\Psi_1\rangle = \alpha|\leftrightarrow_1\rangle + \beta|\Updownarrow_1\rangle, \quad (6)$$

and the Bell-states are defined as

$$\begin{aligned} |\Psi_{12}^{(\pm)}\rangle &= \frac{1}{\sqrt{2}} (|\leftrightarrow_1\rangle|\Updownarrow_2\rangle \pm |\Updownarrow_1\rangle|\leftrightarrow_2\rangle), \\ |\Phi_{12}^{(\pm)}\rangle &= \frac{1}{\sqrt{2}} (|\leftrightarrow_1\rangle|\leftrightarrow_2\rangle \pm |\Updownarrow_1\rangle|\Updownarrow_2\rangle) \end{aligned} \quad (7)$$

in this case.

It is straightforward to see that carrying out a measurement of the (7) Bell-states on the state to be teleported and one branch of the EPR-pair, the other branch of the pair is left in a unitary transform of the state to be teleported [13]. If we know the result of the measurement, the inverse of the unitary transformation can be carried out, and the teleportation is complete.

The interferometric analysis is a similar process to that of one-photon polarization-states [14]. It has been shown, that a beam splitter, combined with two polarization analyzers (as seen in Fig. 3), and four detectors is capable of measuring the (7) Bell-states, and distinguish between the two Ψ states [15]. To illustrate the point, see Fig. 4. The behavior of the Bell-state detector in Fig. 3 is summarized in Table 1.

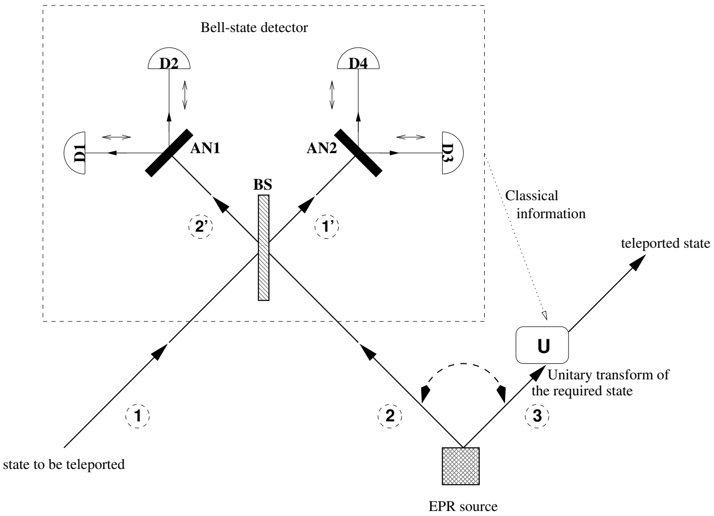

Fig. 3. Teleportation scheme. The Bell-detector consists of the BS beam-splitter, AN1 and AN2 polarization analyzers, and the D1-D4 photon-detectors. The U box inverts the unitary transformation determined by the classical information

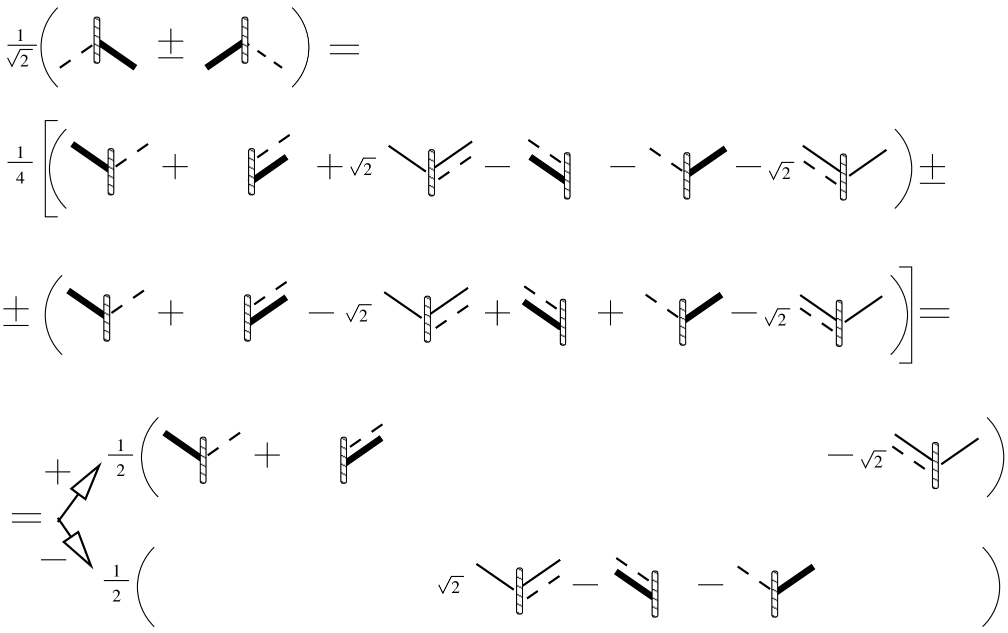

Fig. 4. Measurement of the $|\Psi^{(\pm)}\rangle$ Bell-states using a beam splitter, that imparts no phase shift on the reflected beam, i.e. produces a minus sign to the probability amplitude of each beam reflected from its left side. In this "equation", the symbols refer to photon-number eigenstates at the ports of the beam-splitter. Thin lines correspond to $\updownarrow$, thick lines to $\Updownarrow$ and dashed lines to $\leftrightarrow$ states. The output ports are the upper two ports on each beam-splitter. It is shown in the figure, that the two Ψ states cause different output.

Input state	Event
$\vert\Psi^{(-)}\rangle$	D1D2, D1D4, D2D3D4
$\vert\Psi^{(+)}\rangle$	D2D3, D3D4, D1D2D4
$\vert\Phi^{(+)}\rangle$ or $\vert\Phi^{(-)}\rangle$	D1,D2,D3,D4,D2D4

Table 1. Behavior of the Bell-state detector. The symbols in one term annotate the (coincidently) clicking detectors, while the other detectors keep quiet.

In this way, a three-particle generalization of the teleportation scheme for polarization state teleportation (realized in the Innsbruck experiment) is presented, in which the energy of the teleportable states can differ.

Acknowledgments This research was supported by the National Research Found of Hungary (OTKA) under Contracts T023777 and F019232.

References

[1] D.M. Greenberger, M.A. Horne, A. Zeilinger: *Physics Today* **46**, No. 8 (1993) 22

[2] D.M. Greenberger, M.A. Horne, A. Zeilinger: *Am. J. Phys.* **58** (1990) 1131; T.E. Keller, M.H. Rubin, Y. Shih, L. Wu: *Phys. Rev. A* **57** (1998) 2076

[3] K. Banaszek, P.L. Knight: *Phys. Rev. A* **55** (1997) 2368

[4] J. Janszky, P. Adam, Z. Kis: *Progress of Physics* **46** (1998) 829-835

[5] T. Feilbinger, S. Schiller, J. Mlynek: *Phys. Rev. Lett.* **80** (1998) 492

[6] M. Hillery: *Phys. Rev. A* **42** (1990) 498; and references therein

[7] R. Tanaś, Ts. Gantsog: *Phys. Rev. A* **45** (1992) 5031

[8] G. Drobný, I. Jex: *Phys. Rev. A* **45** (1992) 4897

[9] See e.g. Ref. [7] and references therein.

[10] A. Bandilla, G. Drobný, I. Jex: *Phys. Rev. A* **53** (1996) 507; V. Bužek, G. Drobný: *Phys. Rev. A* **47** (1993) 1237

[11] P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, Y. Shih: *Phys. Rev. Lett.* **75** (1995) 4337

[12] D. Bouvmeester, J. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger: *Nature* **390** (1997) 575

[13] C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters: *Phys. Rev. Lett.* **70** (1993) 1895

[14] H. Weinfurter: *Europhys. Lett.* **25** (8) (1994) 559; S.L. Braunstein, A. Mann: *Phys. Rev. A* **51** (1995) R1727; M. Michler, K. Mattle, H. Weinfurter, A. Zeilinger: *Phys. Rev. A* **53** (1996) R1209; K. Mattle, H. Weinfurter, P. G. Kwiat, A. Zeilinger: *Phys. Rev. Lett.* **76** (1996) 4656

[15] M. Koniorczyk, J. Janszky, Z. Kis: *Phys. Lett. A* (1999) to appear