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QUANTUM STATISTICS OF TWO COUPLED DOWN-CONVERTORS. PART I*

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The quantum-statistical properties of light beams in a directional symmetric nonlinear coupler composed of two nonlinear waveguides operating by the down-conversion processes are examined. By means of short-length approximation non-classical behaviour of single and compound modes in such a device is analyzed. Linear and nonlinear mismatches are taken into account.

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1 Introduction

In paper [1] directional asymmetric nonlinear coupler was discussed composed of a linear waveguide and a nonlinear waveguide operating by the second-harmonic generation from the point of view of all-optical switching. Quantum-statistical properties of such a device were examined using short-length approximation in [2] and the similar research for two coupled nonlinear waveguides was done in [3]. Paper [4] is devoted to the quantum-statistical properties of modes in symmetric coupler using short-length approximation in higher powers with the help of symbolic computations. Properties of a directional coupler involving parametric amplification are studied in [5].

This paper is devoted to the quantum-statistical properties of modes interacting in a directional symmetric nonlinear coupler composed of two nonlinear waveguides operating by a down-conversion processes. The conditions for non-classical behaviour of light in single and compound modes, i.e. squeezing of vacuum fluctuations and sub-Poissonian photon statistics, are analyzed. The influence of linear and nonlinear mismatches on the generation of non-classical light was investigated. The same model is investigated in [6] using the strong-pumping approximation.

In Section 2 the Heisenberg equations of motion is derived, Section 3 is devoted to short-length approximation, Section 3.1 deals with quadrature fluctuations for single and compound modes, Section 3.2 treats the fourth-order normal moments. Section 4 provides a discussion of obtained results.

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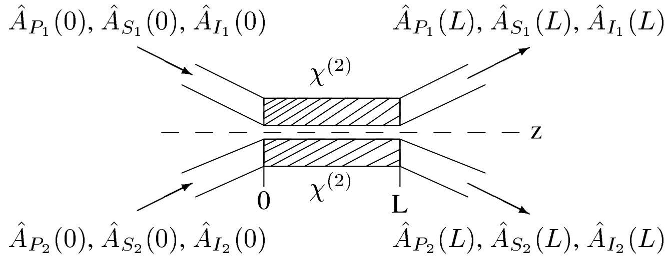
2 Equations of motion

The directional coupler under discussion is composed of two nonlinear waveguides operating by the down-conversion processes in a symmetric arrangement as is outlined in the following figure:



In this coupler a linear coupling between pump, signal and idler fields is considered. The lossless and nondispersive media are assumed. Far from resonant frequencies and assuming directional propagation along z-axis, such a device can be described by the following effective momentum operator:

$$\hat{G} = \hat{G}_1 + \hat{G}_2 + \hat{G}_{int}, \tag{1}$$

where are

$$\begin{aligned} \hat{G}_i &= \hbar \sum_{j=P_i,S_i,I_i} k_j \hat{A}_j^\dagger \hat{A}_j + \hbar \left(\Gamma_i \hat{A}_{P_i} \hat{A}_{S_i}^\dagger \hat{A}_{I_i}^\dagger + \text{h.c.} \right) \quad \text{i=1,2}, \\ \hat{G}_{int} &= \hbar (\kappa_P \hat{A}_{P_1} \hat{A}_{P_2}^\dagger + \kappa_S \hat{A}_{S_1} \hat{A}_{S_2}^\dagger + \kappa_I \hat{A}_{I_1} \hat{A}_{I_2}^\dagger + \text{h.c.}), \end{aligned} \tag{2}$$

and $\hat{A}_j\,(\hat{A}_j^\dagger)$ is annihilation (creation) operator in the j-th mode, i.e. in pumping modes (P_1, P_2), signal modes (S_1, S_2) and idler modes (I_1, I_2). The corresponding wavevectors along the z-axis of propagation are k_{P_1}, k_{P_2}, k_{S_1}, k_{S_2}, k_{I_1} and k_{I_2}. Linear and nonlinear coupling constants are denoted as κ_P, κ_S, κ_I and Γ_1, Γ_2, respectively. The linear coupling constants describe energy exchange between waveguides by means of evanescent waves and the nonlinear coupling constants, which are proportional to the quadratic susceptibility $\chi^{(2)}$, determine the efficiency of nonlinear processes. The symbol $\hbar$ denotes the reduced Planck constant and h.c. represents Hermitian conjugate terms.

The momentum operator $\hat{G}$ in (1) reflects two symmetries of this model. The first symmetry is based on the exchange of subscripts which denote waveguide (i.e. 1 by 2 and vice versa) with simultaneous exchange of linear coupling constants of their complex conjugate ones. The second symmetry consists of the exchange of subscripts between signal and idler modes (i.e. S by I and vice versa). These symmetries have to be conserved in the spatial development of operators and in all quantum statistical quantities. Therefore the results concerning the pump and signal modes in the first waveguide are only mentioned. From the above-mentioned symmetries the remaining results can be obtained.

Quantum spatial development of an operator $\hat{A}$ is described by the Heisenberg equation $i\hbar\, d\hat{A}/dz = [\hat{G}, \hat{A}]$, where z is a spatial variable along the direction of propagation, $\hat{G}$ is above introduced momentum operator and [,] denotes a commutator. Here z is to be understood as a convenient time scale rather then spatial coordinate.

Using the momentum operator (1), we obtain the Heisenberg equations of motion in the form:

$$\begin{aligned}\frac{d\hat{A}_{P_1}}{dz} &= ik_{P_1}\hat{A}_{P_1} + i\kappa_P^*\hat{A}_{P_2} + i\Gamma_1^*\hat{A}_{S_1}\hat{A}_{I_1}, \\ \frac{d\hat{A}_{S_1}}{dz} &= ik_{S_1}\hat{A}_{S_1} + i\kappa_S^*\hat{A}_{S_2} + i\Gamma_1\hat{A}_{P_1}\hat{A}_{I_1}^\dagger. \end{aligned} \tag{3}$$

To eliminate the fast oscillations of the fields we can substitute

$$\hat{A}_{P_1}(z) = \hat{B}_{P_1}(z)\exp(ik_P z), \quad \hat{A}_{S_1}(z) = \hat{B}_{S_1}(z)\exp(ik_S z), \tag{4}$$

where $2k_P = k_{P_1} + k_{P_2}$ and $2k_S = k_{S_1} + k_{S_2}$ into (3) and the following set of nonlinear equations is obtained:

$$\begin{aligned}\frac{d\hat{B}_{P_1}}{dz} &= i\frac{\Delta k_P}{2}\hat{B}_{P_1} + i\kappa_P^*\hat{B}_{P_2} + i\Gamma_1^*\hat{B}_{S_1}\hat{B}_{I_1}\exp(i\Delta kz), \\ \frac{d\hat{B}_{S_1}}{dz} &= i\frac{\Delta k_S}{2}\hat{B}_{S_1} + i\kappa_S^*\hat{B}_{S_2} + i\Gamma_1\hat{B}_{P_1}\hat{B}_{I_1}^\dagger\exp(-i\Delta kz), \end{aligned} \tag{5}$$

where $\Delta k_P = k_{P_1} - k_{P_2}$, $\Delta k_S = k_{S_1} - k_{S_2}$ and $\Delta k = k_S + k_I - k_P$ are the linear and nonlinear mismatches, respectively.

3 Short-length approximation

Equations (5) can be solved, using the short-length approximation, up to z^2 in the form:

$$\begin{aligned}\hat{B}_{P_1}(z) &= \left(1 + i\frac{\Delta k_P}{2}z - \frac{1}{2}\left[\left(\frac{\Delta k_P}{2}\right)^2 + |\kappa_P|^2\right]z^2\right)\hat{A}_{P_1}(0) + i\kappa_P^* z\hat{A}_{P_2}(0) \\ &\quad + \Gamma_1^*\left(iz + \frac{\Delta k_{211}}{2}z^2\right)\hat{A}_{S_1}(0)\hat{A}_{I_1}(0) - \frac{1}{2}\Gamma_1^*\kappa_I^* z^2\hat{A}_{S_1}(0)\hat{A}_{I_2}(0) \\ &\quad - \frac{1}{2}\Gamma_1^*\kappa_S^* z^2\hat{A}_{S_2}(0)\hat{A}_{I_1}(0) - \frac{1}{2}\Gamma_2^*\kappa_P^* z^2\hat{A}_{S_2}(0)\hat{A}_{I_2}(0) \\ &\quad - \frac{1}{2}|\Gamma_1|^2 z^2\hat{A}_{P_1}(0)\left[\hat{A}_{S_1}^\dagger(0)\hat{A}_{S_1}(0) + \hat{A}_{I_1}^\dagger(0)\hat{A}_{I_1}(0) + 1\right], \\ \hat{B}_{S_1}(z) &= \left(1 + i\frac{\Delta k_S}{2}z - \frac{1}{2}\left[\left(\frac{\Delta k_S}{2}\right)^2 + |\kappa_S|^2\right]z^2\right)\hat{A}_{S_1}(0) + i\kappa_S^* z\hat{A}_{S_2}(0) \\ &\quad + \Gamma_1\left(iz - \frac{\Delta k_{121}}{2}z^2\right)\hat{A}_{P_1}(0)\hat{A}_{I_1}^\dagger(0) + \frac{1}{2}\Gamma_1\kappa_I z^2\hat{A}_{P_1}(0)\hat{A}_{I_2}^\dagger(0) \\ &\quad - \frac{1}{2}\Gamma_2\kappa_S^* z^2\hat{A}_{P_2}(0)\hat{A}_{I_2}^\dagger(0) - \frac{1}{2}\Gamma_1\kappa_P^* z^2\hat{A}_{P_2}(0)\hat{A}_{I_1}^\dagger(0) \\ &\quad + \frac{1}{2}|\Gamma_1|^2 z^2\hat{A}_{S_1}(0)\left[\hat{A}_{P_1}^\dagger(0)\hat{A}_{P_1}(0) - \hat{A}_{I_1}^\dagger(0)\hat{A}_{I_1}(0)\right], \end{aligned} \tag{6}$$

where $\hat{A}_j(0) = \hat{B}_j(0)$ $(\hat{A}_j^\dagger(0) = \hat{B}_j^\dagger(0))$ are initial values of anihilation (creation) operators and $\Delta k_{ijk} = k_{P_i} - k_{S_j} - k_{I_k}$ for $i, j, k = 1, 2$ is nonlinear mismatch.

From the point of view of the quantum-statistical properties, the short-length solutions provide us with a good approximation for one-pass propagation under the assumption that $|\Gamma_1|z$, $|\Gamma_2|z \ll 1$.

3.1 Quadrature fluctuations

To detect quadrature fluctuations, one can adopt the homodyne detection technique, where fast oscillations of a field are eliminated by means of local oscillator and the following principal squeeze variances for the j-th single mode ([7], p.240)

$$\lambda_j = 1 + 2[\langle \Delta\hat{B}_j^\dagger \Delta\hat{B}_j\rangle - |\langle(\Delta\hat{B}_j)^2\rangle|], \tag{7}$$

and (jk)-th compound mode ([7], p.259)

$$\begin{aligned}\lambda_{jk} &= 1 + \langle \Delta\hat{B}_j^\dagger \Delta\hat{B}_j\rangle + \langle \Delta\hat{B}_k^\dagger \Delta\hat{B}_k\rangle + 2\mathrm{Re}\langle \Delta\hat{B}_j^\dagger \Delta\hat{B}_k\rangle \\ &\quad -|\langle(\Delta\hat{B}_j)^2\rangle + \langle(\Delta\hat{B}_k)^2\rangle + 2\langle\Delta\hat{B}_j\Delta\hat{B}_k\rangle| \qquad j \neq k,\end{aligned} \tag{8}$$

where $j, k = P_1, P_2, S_1, S_2, I_1, I_2$ are measured. Squeezing of vacuum fluctuations occurs, if $\lambda < 1$ in the corresponding mode.

Substituting the solution (6) into definition (7) and by means of the standard averaging procedure of normal order operators in the incident coherent state, we obtain

$$\lambda_{P_1} = 1, \quad \lambda_{S_1} = 1 + 2|\Gamma_1|^2|\alpha_{P_1}|^2 z^2. \tag{9}$$

We can see that modes P_1 and P_2 remain coherent during propagation in this approximation. In contrast, quadrature fluctuations of modes S_1, S_2, I_1 and I_2 exceed those of the coherent state. These results are valid for both stimulated and spontaneous processes.

In the same way, we obtain the principal squeeze variances for the compound modes:

$$\begin{aligned}\lambda_{P_1P_2} &= 1, \quad \lambda_{S_1S_2} = 1 + \left(|\Gamma_1|^2|\alpha_{P_1}|^2 + |\Gamma_2|^2|\alpha_{P_2}|^2\right) z^2, \\ \lambda_{P_1S_2} &= 1 + |\Gamma_2|^2|\alpha_{P_2}|^2 z^2, \quad \lambda_{P_1S_1} = 1 + |\Gamma_1|^2|\alpha_{P_1}|\left(|\alpha_{P_1}| - |\alpha_{S_1}|\right) z^2, \\ \lambda_{S_1I_1} &= 1 - |\Gamma_1\left(2i\alpha_{P_1}z - \Delta k_{122}\alpha_{P_1}z^2 - \kappa_P^*\alpha_{P_2}z^2\right) - |\Gamma_1|^2\alpha_{S_1}\alpha_{I_1}z^2| \\ &\quad +2|\Gamma_1|^2|\alpha_{P_1}|^2 z^2, \\ \lambda_{S_1I_2} &= 1 + \left(|\Gamma_1|^2|\alpha_{P_1}|^2 + |\Gamma_2|^2|\alpha_{P_2}|^2 - |\Gamma_1\kappa_I\alpha_{P_1} + \Gamma_2\kappa_S^*\alpha_{P_2}|\right) z^2.\end{aligned} \tag{10}$$

The compound mode (P_1P_2) remain coherent during propagation. The quadrature fluctuations of modes (P_1S_2), (P_2S_1), (P_1I_2), (P_2I_1), (S_1S_2) and (I_1I_2) always increase during propagation. The squeezing of vacuum fluctuations in (P_1S_1) and (P_1I_1) can arise, when the amplitudes of input fields satisfy the following conditions $|\alpha_{S_1}| > |\alpha_{P_1}|$ and $|\alpha_{I_1}| > |\alpha_{P_1}|$, respectively. The similar conclusions are valid with respect to the modes (P_2S_2) and (P_2I_2) when the corresponding inequalities are satisfied. The modes (S_1I_1) and (S_2I_2) can be squeezed depending on the nonlinear mismatch, phases of incident fields and phases of coupling constants. For sufficiently small values of $|\Gamma_1|z$ and $|\Gamma_2|z$, it approximately holds that $\lambda_{S_1I_1} = 1 - 2|\Gamma_1\alpha_{P_1}|z$ and $\lambda_{S_2I_2} = 1 - 2|\Gamma_2\alpha_{P_2}|z$, respectively, and the squeezing of vacuum fluctuations occurs in these modes. If we consider the waveguides to be identical ($\Gamma_1 = \Gamma_2$) together with input condition $\alpha_{P_1} = \alpha_{P_2}$, then in the case, where the linear coupling dominates above the nonlinear coupling (i.e. $|\kappa_I + \kappa_S{}^*| > 2|\Gamma_1 \alpha_{P_1}|$), the vacuum noise enlarged by nonlinear interaction is suppressed due to the linear interaction and the squeezing of modes $(S_1 I_2)$ and $(I_1 S_2)$ is obtained.

3.2 Fourth-order normal moments

One of the most important characteristics of photon statistics are the fourth-order normal moments ([7], p.268) defined for mode j as:

$$\langle(\Delta W_j(z))^2\rangle_{\mathcal{N}} = \langle \hat{B}_j^{\dagger 2}(z)\hat{B}_j^2(z)\rangle - \langle \hat{B}_j^{\dagger}(z)\hat{B}_j(z)\rangle^2. \tag{11}$$

Non-classical behaviour is exhibited when $\langle(\Delta W_j(z))^2\rangle_{\mathcal{N}} < 0$. For the compound mode (jk) the correlation of fluctuations of integrated intensities ([7], p.269) is of interest:

$$\langle\Delta W_j(z)\Delta W_k(z)\rangle_{\mathcal{N}} = \langle \hat{B}_j^{\dagger}(z)\hat{B}_k^{\dagger}(z)\hat{B}_j(z)\hat{B}_k(z)\rangle - \langle \hat{B}_j^{\dagger}(z)\hat{B}_j(z)\rangle\langle \hat{B}_k^{\dagger}(z)\hat{B}_k(z)\rangle, \tag{12}$$

for $j, k = P_1, P_2, S_1, S_2, I_1, I_2$ and $j \neq k$, where $W_j(z)$ is the corresponding integrated intensity and the subscript $\mathcal{N}$ denotes normal ordering of field operators [6].

Considering incident beams being in the coherent states as above and substituting (6) into (11), we calculate the following variances:

$$\langle(\Delta W_{P_1}(z))^2\rangle_{\mathcal{N}} = 0, \quad \langle(\Delta W_{S_1}(z))^2\rangle_{\mathcal{N}} = 2|\Gamma_1|^2|\alpha_{P_1}|^2|\alpha_{S_1}|^2 z^2. \tag{13}$$

It is evident that modes P_1 and P_2 remain coherent during propagation as in the case of the second-order moments discussed in Sec. 3.1. On the other hand modes S_1, S_2, I_1 and I_2 are super-Poissonian. If the process is completely spontaneous ($\alpha_{P_1}, \alpha_{P_2} \neq 0; \alpha_{S_1}, \alpha_{S_2}, \alpha_{I_1}, \alpha_{I_2} = 0$) all modes have Poissonian photon statistics.

For the compound modes we can derive the following mutual correlations of fluctuations of integrated intensities:

$$\begin{aligned}
\langle\Delta W_{P_1}\Delta W_{P_2}\rangle_{\mathcal{N}} &= \langle\Delta W_{P_1}\Delta W_{S_2}\rangle_{\mathcal{N}} = \langle\Delta W_{S_1}\Delta W_{S_2}\rangle_{\mathcal{N}} = 0,\\
\langle\Delta W_{P_1}\Delta W_{S_1}\rangle_{\mathcal{N}} &= -|\Gamma_1|^2|\alpha_{P_1}|^2|\alpha_{S_1}|^2 z^2,\\
\langle\Delta W_{S_1}\Delta W_{I_2}\rangle_{\mathcal{N}} &= -\frac{1}{2}\left(\Gamma_2\kappa_S^*\alpha_{S_1}^*\alpha_{P_2}\alpha_{I_2}^* + \Gamma_1^*\kappa_I^*\alpha_{P_1}^*\alpha_{S_1}\alpha_{I_2} + \text{c.c.}\right) z^2,\\
\langle\Delta W_{S_1}\Delta W_{I_1}\rangle_{\mathcal{N}} &= \left(i\Gamma_1\alpha_{P_1}\alpha_{S_1}^*\alpha_{I_1}^* + \text{c.c.}\right) z\\
&\quad + \left(|\Gamma_1|^2\left[|\alpha_{P_1}|^2\left(2|\alpha_{S_1}|^2 + 2|\alpha_{I_1}|^2 + 1\right) - |\alpha_{S_1}|^2|\alpha_{I_1}|^2\right]\right.\\
&\quad - \left[1/2\left(\Gamma_1^*\kappa_P\alpha_{S_1}\alpha_{I_1}\alpha_{P_2}^* + \Gamma_1\Delta k_{111}\alpha_{P_1}\alpha_{S_1}^*\alpha_{I_1}^*\right)\right.\\
&\quad \left.\left. - \ \Gamma_1\alpha_{P_1}\left(\kappa_S\alpha_{I_1}^*\alpha_{S_2}^* + \kappa_I\alpha_{S_1}^*\alpha_{I_2}^*\right) + \text{c.c.}\right]\right) z^2,
\end{aligned} \tag{14}$$

where c.c. denotes complex conjugation. The compound modes (P_1P_2), (P_1S_2), (P_1I_2), (S_1P_2), (S_1S_2), (I_1P_2) and (I_1I_2) are always uncorrelated. Modes (P_1S_1), (P_1I_1), (P_2S_2) and (P_2I_2) are anticorrelated when the modes S_1, I_1, S_2 and I_2 are stimulated, respectively. Compound modes (S_1I_2) and (I_1S_2) can be correlated or anticorrelated depending on the phases of the incident fields and coupling constants. For example, if the following phase condition $\arg(\Gamma_2\kappa_S^*\alpha_{S_1}^*\alpha_{P_2}\alpha_{I_2}^*) = \arg(\Gamma_1^*\kappa_I^*\alpha_{P_1}^*\alpha_{S_1}\alpha_{I_2}) = 0$ is satisfied, maximum anticorrelation in the compound mode $(S_1 I_2)$ is attained. Using the above-mentioned symmetry we obtain the similar condition for maximum anticorrelation in mode $(I_1 S_2)$. As for compound modes $(S_1 I_1)$ and $(S_2 I_2)$ correlations of fluctuations can be supported by linear coupling between waveguides and nonlinear mismatch. For partially stimulated process ($\alpha_{P_1}, \alpha_{P_2} = 0$, $\alpha_{S_1}, \alpha_{I_1}, \alpha_{S_2}, \alpha_{I_2} \neq 0$) the anticorrelations are exhibited in the compound modes $(S_1 I_1)$ and $(S_2 I_2)$. Assuming small values of $|\Gamma_1|z$, we can omit terms containing z^2 and anticorrelation occurs in compound mode $(S_1 I_1)$, when $\arg(\Gamma_1 \alpha_{P_1} \alpha_{S_1}^* \alpha_{I_1}^*) = \frac{\pi}{2}$. The similar conclusion is valid for mode $(S_2 I_2)$. In the spontaneous process all compound modes are uncorrelated in this approximation, except for the compound modes $(S_1 I_1)$ and $(S_2 I_2)$ which are correlated.

4 Discussion

The quantum-statistical properties of the directional symmetric nonlinear coupler composed of two nonlinear waveguides based on the down-conversion processes were discussed. In the framework of short-length approximation up to the second order the solution of equations of motion was derived. Principal squeeze variances for single and compound modes, variances of integrated intensities and correlations of fluctuations of integrated intensities were obtained. As for single-mode characteristics the coupler behaves like two independent waveguides in this approximation. This nonlinear coupler is more interesting from the point of view of two-mode characteristics, how it was shown above.

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