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QUANTUM CLONING AND SIGNALING*

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We discuss the close connections between cloning of quantum states and superluminal signaling. We present an optimal universal cloning machine based on stimulated emission recently proposed by us. As an instructive example, we show how a scheme for superluminal communication based on this cloning machine fails.

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1 Introduction

To our knowledge, the discussion about the cloning of quantum systems started with a paper by Herbert [1], where he proposed a method for superluminal communication. His scheme made use of pairs of entangled particles shared by the two parties that would like to communicate (Alice and Bob), and of what he called idealized laser tubes, which would today be called universal cloning machines. The basic idea of his proposal was the following. Alice and Bob each have one member of a pair of entangled particles, e.g. photons described by the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|VH\rangle - |HV\rangle), \tag{1}$$

where V and H denote vertical and horizontal polarization respectively. Alice can measure the polarization of her particle either in the basis $|V\rangle, |H\rangle$ or in the basis $|P\rangle, |M\rangle$, where $|P\rangle = \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle)$ and $|M\rangle = \frac{1}{\sqrt{2}}(|V\rangle - |H\rangle)$. If Alice measures in the V/H basis and finds $|V\rangle$ ($|H\rangle$), Bob's photon is reduced to $|H\rangle$ ($|V\rangle$), while if she measures in the P/M basis (results $|P\rangle$ and $|M\rangle$) Bob's photon is reduced to the corresponding states in that basis ($|M\rangle$ and $|P\rangle$ respectively). Although the states on his side are therefore different depending on Alice's choice of basis, a priori this does not allow Bob to know her choice because he cannot discriminate $|V\rangle$ from $|P\rangle$ by a single measurement (having only a single copy). But imagine that he has a machine that can produce an arbitrary number of copies of any one-photon state, or at least of the states $|V\rangle$ and $|P\rangle$. This allows Bob to discriminate the two states and

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in this way Alice's two choices of basis. If Bob's copier works fast enough, this establishes a superluminal communication channel. Herbert proposed stimulated emission as a possible working principle for his copying machine. He was aware of the fact that spontaneous emission could be a problem, but thought that it would not be fatal for the scheme.

On the other hand, it is possible to show in a general way that superluminal signaling (signaling between systems whose time development and projection operators commute) is not possible in quantum mechanics [2]. One way of explaining why Alice cannot signal to Bob is the following. Bob does not know which result Alice got, so his photon is described by a density matrix $\frac{1}{2}(|V\rangle\langle V|+|H\rangle\langle H|)$, if she measured in the V/H basis, and $\frac{1}{2}(|P\rangle\langle P|+|M\rangle\langle M|)$, if she measured in the P/M basis. Of course, these two density matrices are identical, so there is no way for him to tell what she did. This also shows that a cloner doesn't help, as long as it is describable by quantum mechanics, because any quantum device will, fed by identical input density matrices, produce identical outputs.

Apparently triggered by Herbert's paper, Wootters and Zurek [3] showed that perfect cloning of arbitrary quantum systems (or even only of the states needed for Herbert's scheme) is not possible. This is a consequence of the linearity of quantum mechanics [4]. About 14 years later, Bužek and Hillery [5] considered the non-perfect copying of quantum systems. They constructed a universal quantum copying machine that produces two non-perfect clones of an input qubit, whose fidelity does not depend on the input. Since then, more general cloning machines producing M clones starting from N qubits have been constructed [6], and bounds for the possible fidelity of the clones have been derived [7], showing that the machines constructed by Bužek and Hillery and by Gisin and Massar are optimal. The devices proposed by these authors are based on quantum gates. Recently we have shown that optimal universal cloning can be realized via stimulated emission [8]. One can say that in this scheme the deteriorating effect of spontaneous emission is exactly as large as to reduce the fidelity of the clones from the ideal value 1 to the maximum allowed value.

Given the above-mentioned results, one does not expect optimal universal cloning machines to make faster-than-light communication possible. For 1 to 2 cloners, Gisin [9] has even shown that the maximum possible fidelity of the clones can also be derived from the no-signaling condition. Whether this result can be generalized is, to our knowledge, an open problem.

Although the impossibility of superluminal signaling is well established, the fact that cloning with high fidelity does not make it possible can still be quite counter-intuitive in concrete examples. In the following we will briefly present our optimal universal cloning machine based on parametric down-conversion. It is more fully described elsewhere [8], together with another scheme that is also based on stimulated emission. Then we will show how the most obvious scheme for superluminal signaling using this device fails. Although this fact in itself is by no means surprising, we still feel that it is instructive to see how the impossibility arises in a concrete case.

2 An Optimal Cloning Machine

In pulsed parametric down-conversion (PDC) a strong light pulse is sent through a crystal. There is a certain (very low) probability for a photon from this so called pump pulse to decay into two photons such that energy and crystal momentum are conserved. It is possible to choose

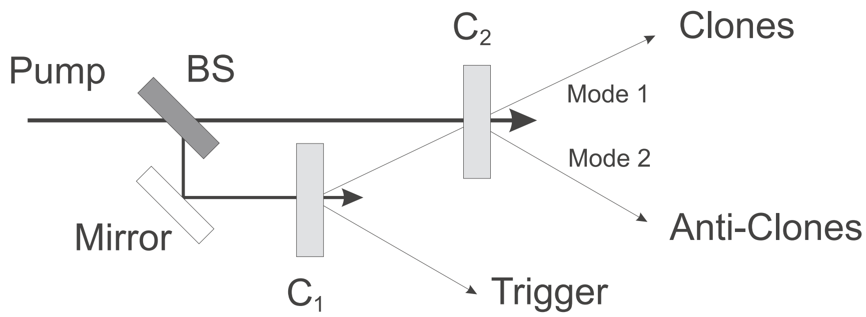

Fig. 1. Setup for cloning by parametric down-conversion. The pump-pulse is split at the beamsplitter BS. The smaller part of the pump pulse hits the first crystal C_1, where photon pairs are created with a certain rate. Consider the case where exactly one pair is created. The photon created in the lower mode is used as a trigger. The upper photon is the system to be cloned. This photon is directed towards the second crystal C_2, as is the rest of the pump pulse, where it stimulates emission of photons of the same polarization along the same direction. The path lengths are adjusted in such a way that the DC-photon and the pump pulse reach C_2 simultaneously. The photons in mode 1 can be considered as clones of the incoming photon. Mode 2 is a negative image of mode 1 (apart from the incoming photon) and can be seen as realization of an optimal universal NOT-gate. This particular setup is chosen in order to make the principle transparent (see also ref. [10]). In practice one would probably use a different setup, where the pump-pulse is not split into two parts, but reflected so that it propagates through the crystal twice, and also the down-conversion photons that are to be cloned are reflected in the same way.

two conjugate directions such that the creation of photon pairs along these two directions is described by a Hamiltonian

$$H = \gamma(a_{V1}^\dagger a_{H2}^\dagger - a_{H1}^\dagger a_{V2}^\dagger) + h.c., \tag{2}$$

where $a_{V1}^\dagger$ is the creation operator for a photon with polarization V propagating along direction 1 etc. The coupling constant and the intensity of the classical pump pulse are contained in γ. See reference [8] for a more detailed discussion of the conditions for this description to be valid.

The Hamiltonian H is invariant under general common $SU(2)$ transformations of the polarization vectors $(a_V^\dagger, a_H^\dagger)$ for modes 1 and 2, while a phase transformation will only change the phase of γ. This makes our cloner universal, i.e. its performance is polarization independent. Consider a photon of arbitrary polarization coming in. The Hamiltonian looks exactly the same in the new polarization basis defined by this photon, so also the final state will look the same. Therefore it is sufficient to analyze the "cloning" process in one basis, e.g. for an incoming N-photon state $|\psi_i\rangle = (a_{V1}^\dagger)^N|0\rangle$.

Making use of the disentangling theorem [13] one finds that the final state is given by (cf. [10])

$$|\psi_f\rangle = e^{-iHt}|\psi_i\rangle = K\sum_{k=0}^{\infty}(-i\Gamma)^k\sqrt{\binom{k+N}{N}}|k+N\rangle_{V1}|k\rangle_{H2}\sum_{l=0}^{\infty}(i\Gamma)^l|l\rangle_{H1}|l\rangle_{V2} \tag{3}$$

where $\Gamma = \tanh\gamma t$ and K is a normalizing factor.

The component of this state that has a fixed number M of photons in mode 1, is proportional to

$$\sum_{l=0}^{M-N}(-1)^l\sqrt{\binom{M-l}{N}}|M-l\rangle_{V1}|l\rangle_{H1}|l\rangle_{V2}|M-N-l\rangle_{H2}. \quad (4)$$

This is identical to the state produced by the unitary transformation written down in [14] which can be seen as a special version of the Gisin-Massar cloners [6] that implements optimal universal cloning and the optimal universal NOT-gate at the same time. The M photons in mode 1 are the clones, while the $M-N$ photons in mode 2 are the output of the universal NOT-gate, the "anti-clones". This means that the setup of Fig. 1 works as an ensemble of optimal universal cloning (and universal NOT) machines, producing different numbers of clones and anti-clones with certain probabilities. Note that each of the modes can be used as a trigger for the other one so that cloning or anti-cloning with a fixed number of output-systems can be realized by post-selection.

To get a feeling for the nature of the state (3), one can calculate the mean numbers of photons in the various modes for the case of one incoming V-polarized photon in mode 1, i.e. an initial state $|\psi_i\rangle = a_{V1}^\dagger|0\rangle$. One finds

$$\begin{aligned} N_{V1}(t) &= 2\sinh^2\gamma t + 1, \\ N_{H2}(t) &= 2\sinh^2\gamma t, \\ N_{H1}(t) &= N_{V2}(t) = \sinh^2\gamma t. \end{aligned} \quad (5)$$

Roughly speaking, the effect of stimulated emission is a factor of two between the numbers of V (right) and H (wrong) photons in mode 1. If there is no photon coming in, all the mean numbers are equal to $\sinh^2\gamma t$.

3 A scheme that fails

Now let us discuss the question of signaling for this particular cloning machine. Let Alice and Bob proceed exactly as outlined in the introduction. Alice measures the polarization of her photon in either the V/H or the P/M basis, Bob feeds his photon into our optimal cloner. Shouldn't this allow him to decide in which basis Alice measured? If a photon with polarization V is fed in, the cloner will produce a state with a clear excess of V-polarized photons in mode 1. In the same way, an H-photon will result in an excess of H-polarized photons. On the other hand, P and M photons coming in will result in states where V- and H-polarized photons occur with equal probability (this is clear from symmetry considerations, and can also be easily verified directly) . Shouldn't it be possible to detect this difference and in this way infer the basis Alice used? Bob would just have to perform a polarization analysis in the V/H basis of all photons coming out from his cloner. If he finds a clear excess of one polarization, this would be an indication that Alice measured in the V/H basis, if he finds similar numbers of V- and H-polarized photons, it would mean that she measured in the P/M basis.

To see why this scheme doesn't work, define $\Delta N = N_V - N_H$, which denotes the difference in number between V and H polarized photons in the output of the cloner. The final state [Eq. (3)] is a superposition of states with different numbers of photons and also with different values of ΔN. For an incoming V and an incoming H photon ΔN has probability distributions $p_V(\Delta N)$ and $p_H(\Delta N)$ respectively, where p_V is peaked around a large positive value (namely $\sinh^2\gamma t+1$), and $p_H(\Delta N)=p_V(-\Delta N)$. The probability distributions for ΔN in the case of P or M polarized photons coming in are

$$p_P(\Delta N)=p_M(\Delta N)=\frac{1}{2}(p_V(\Delta N)+p_H(\Delta N)). \tag{6}$$

This follows from the fact that all the terms in the expansion of the final state $|\psi_f\rangle$ for an initial V photon are of the form $|k+1\rangle_{V1}|k\rangle_{H2}|l\rangle_{H1}|l\rangle_{V2}$ (i.e. there are $V1-H2$ and $H1-V2$ pairs plus the initial photon) and are orthogonal to all the terms occuring in the expansion for $|\psi_f\rangle$ for an initial H state (where there are pairs plus an additional $H1$ photon). This means that there is no interference and one simply has to add the probabilities.

Now imagine that Bob chooses some threshold value ΔN_{th}. If he finds $|\Delta N|>\Delta N_{th}$, he assumes that Alice measured in the V/H basis. How big is the error he makes? The probability to find $|\Delta N|>\Delta N_{th}$ if she chose the V/H basis is given by

$$\frac{1}{2}P_V(|\Delta N|>\Delta N_{th})+\frac{1}{2}P_H(|\Delta N|>\Delta N_{th}), \tag{7}$$

where

$$P_{V(H)}(|\Delta N|>\Delta N_{th})=\sum_{|\Delta N|>\Delta N_{th}} p_{V(H)}(\Delta N). \tag{8}$$

The probability to find $|\Delta N|>\Delta N_{th}$ if Alice chose the P/M basis is

$$\frac{1}{2}P_P(|\Delta N|>\Delta N_{th})+\frac{1}{2}P_M(|\Delta N|>\Delta N_{th}), \tag{9}$$

which is exactly identical to (7), because of (6). Thus, somewhat counter-intuitively, events with a high asymmetry in photon numbers are exactly as likely when Alice measures in the P/M basis as when she measures in the V/H basis.

This means that, following this procedure, Bob makes an error in exactly half of the cases, which is the value he would obtain by random guessing; he does not gain any information whatsoever about Alice's basis. The same argument applies, of course, to the attempt to infer from small values of ΔN that Alice was measuring in the P/M basis.

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[4] If Bob had a machine that produces an arbitrary number of copies of the states $|V\rangle$ and $|P\rangle$, this would already allow him to know Alice's basis in 50 percent of the cases. That such a machine cannot exist can also be seen from the requirement of unitarity. It would have to be described by a transformation

$$|V\rangle \otimes |\phi_0\rangle \rightarrow |VV...V\rangle \otimes |\phi_V\rangle$$
$$|P\rangle \otimes |\phi_0\rangle \rightarrow |PP...P\rangle \otimes |\phi_P\rangle,$$

where $|\phi_{0(V,P)}\rangle$ are the initial and final states of the cloning machine. The scalar product of the two initial states is $\frac{1}{\sqrt{2}}$, while the scalar product of the two final states is at most $(\frac{1}{\sqrt{2}})^M$, where M is the number of copies produced. This means that the transformation cannot be unitary.

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