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QUANTUM SUPERPOSITIONS OF TWO COHERENT STATES GENERATION BASED ON A SINGLE-ATOM CONDITIONAL MEASUREMENT

A. Napoli, A. Messina[1]
INFM and MURST
Dipartimento di Scienze Fisiche ed Astronomiche dell'Università di Palermo, via Archirafi 36, 90123 Palermo, Italy

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A new and simple way of engineering quantum superpositions of two coherent states of a single-mode quantized electromagnetic field is presented. Our proposal, developed in the context of micromaser theory, exploits the passage of one atom only through a high-Q bimodal cavity supporting two electromagnetic modes of different frequencies.

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Quite recently quantum superpositions of two coherent states of light have received a great deal of attention from both experimentalists and theoreticians. The interest toward these states stems from the consideration that, due to probability amplitude interference effects [1], they might exhibit nonclassical features pronounced enough to provide almost ideal conditions to test basic aspects of quantum mechanics and to explore the border between classical and quantum description of nature [2]. In particular it has been shown [1] that squeezing, sub-Poissonian photon statistics and oscillations of the photon number distribution emerge from a superposition of coherent states.

In this paper we present a new and simple way of engineering a class of states obtained as quantum superpositions of two coherent states of a single-mode quantized electromagnetic field. Our proposal, developed in the context of micromaser theory, exploits the passage of one atom only through a high-Q ($Q \approx 5 \times 10^{10}$) bimodal cavity supporting two electromagnetic modes of different frequencies ω_1 and ω_2 such that $\omega_1 \approx \omega_2 \approx 10^{10}$Hz. The atom is a Rydberg atom effectively behaving as a two-level atom whose ground state $|-\rangle$ and excited state $|+\rangle$ are energetically separated by ω_0 ($\hbar = 1$). The cavity mode to be manipulated, hereafter also called mode of interest, is that of frequency ω_1 and is initially prepared in a coherent state $|\alpha\rangle$ where $\alpha \in C$. In the context of our procedure the other mode plays an auxiliary role and at $t = 0$ is left in its empty state. Our experimental scheme relies on the assumption $\omega_0 \approx \omega_1 + \omega_2$ so that the atom-field coupling may be adequately described in accordance with the following effective resonant two-photon Hamiltonian model:

$$H = \sum_{\mu=0}^{2} \omega_\mu \alpha_\mu^\dagger \alpha_\mu + (\omega_0 + \beta_2 \alpha_2^\dagger \alpha_2 - \beta_1 \alpha_1^\dagger \alpha_1) S_z + \lambda[\alpha_1 \alpha_2 S_+ + \text{h.c.}] \tag{1}$$

[1]E-mail address: messina@fisica.unipa.it

well-known in literature after Gou [3]. As usual, the Pauli pseudo spin operators S_z and $S_\pm$ describe the atomic internal degrees of freedom and are such that $2S_z|\pm\rangle = \pm|\pm\rangle$ and $S_\pm|\mp\rangle = |\pm\rangle$ whereas α_i ($\alpha_i^\dagger$) is the annihilation (creation) operator relative to the i-th mode of the cavity field.

Each atomic excitation (de-excitation) is accompanied by the simultaneous absorption (emission) of one photon from (into) the mode of interest and one photon from (into) the auxiliary mode. The coupling constant λ measures the strength of the atom-field energy exchanges and, as in a typical two-photon micromaser experiment [4], may be taken of the order of 10^3Hz. The intensity-dependent detuning is characterized by the two Stark parameters β_1 and β_2 here assumed coincident for simplicity and such that $\beta_1 = \beta_2 \approx 10^3$Hz [4].

The Hamiltonian model (1) is exactly solvable and the time evolution of the atom-cavity system has been exactly treated in Ref. [5]. Denote by $|p\rangle$ a generic Fock state of the mode of interest and by $|m\rangle$ a generic eigenstate of $\alpha_2^\dagger\alpha_2$. Ref. [5] shows that the time evolution of the atom-field state $|p\rangle|m=0\rangle|+\rangle \equiv |p,0,+\rangle$ may be cast in the form

$$\exp\left(-iHt\right)|p,0,+\rangle = C_p(t)|p,0,+\rangle + S_p(t)|p+1,1,-\rangle \tag{2}$$

where

$$C_p(t) = \left[\cos\left(\Lambda\left(p+1,1\right)t\right) - ic(p+1,1)\sin\left(\Lambda\left(p+1,1\right)t\right)\right]\exp\left(-i\epsilon_0(p+1,1)t\right), \tag{3}$$

$$S_p(t) = -is(p+1,1)\sin\left(\Lambda\left(p+1,1\right)t\right)\exp\left(-i\epsilon_0(p+1,1)t\right), \tag{4}$$

$$s^2(p+1,1) = \frac{4(p+1)}{(p+2)^2}, \quad \Lambda(p+1,1) = \frac{\lambda}{2}(p+2)^2, \tag{5}$$

$$s^2(p+1,1) + c^2(p+1,1) = 1, \quad \epsilon_0(p+1,1) = \omega_1(p+\frac{1}{2}) + \omega_2. \tag{6}$$

Suppose now that the Rydberg atom is prepared in its excited state before entering the cavity. The initial condition of atom-field system in our experimental scheme may be represented as

$$|\Psi(0)\rangle = |\alpha\rangle|0\rangle|+\rangle \qquad \alpha \in C \tag{7}$$

with the mode of frequency ω_1 in the coherent state

$$|\alpha\rangle = \sum_{p=0}^{\infty} e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^p}{\sqrt{p!}}|p\rangle. \tag{8}$$

Indicate by τ the time spent by the atom inside the cavity and by $|\Psi(\tau)\rangle$ the normalized entangled atom-field state at $t=\tau$. If immediately after leaving the cavity, an appropriate detector of the internal atomic state finds, with a τ-dependent not vanishing probability $P(\tau)$, the atom not excited, then, in view of Eqs. (2), (4) and (8), the bimodal cavity field collapses into the factorized state $|\varphi(\tau)\rangle|1\rangle$. Here

$$|\varphi(\tau)\rangle = -iN(\tau)\sum_{p=0}^{\infty} e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^p}{\sqrt{p!}}S_p(\tau)|p+1\rangle \tag{9}$$

denotes the new normalized state reached by the mode of interest starting from $|\alpha\rangle$ as consequence of both the flight of the atom trough the cavity and the subsequent detection of the Rydberg atom in its ground state $|-\rangle$. The positive normalization constant $N(\tau)$ appearing in Eq. (9) may be written down as (the phase factor i appears for convenience)

$$N(\tau) = \left| \sum_{p=0}^{\infty} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^p}{\sqrt{p!}} |S_p(\tau)|^2 \right|^{-\frac{1}{2}} \equiv [P(\tau)]^{-\frac{1}{2}} \tag{10}$$

and, of course, is mathematically well defined only when $P(\tau) \neq 0$ that is, in view of Eq. (5), when $\lambda\tau \neq 2k\pi$ with $k \in Z$. In order to gain more insight on the properties of the state described by Eq. (9), we exploit the exponential Eulerian representation of the sinusoidal factor appearing in the expression (4), getting:

$$|\varphi(\tau)\rangle = A \left[\sum_{q=0}^{\infty} D_q \left(\alpha e^{i\gamma_-}\right) |q\rangle - e^{-i\vartheta} \sum_{q=0}^{\infty} D_q \left(\alpha e^{i\gamma_+}\right) |q\rangle \right] \tag{11}$$

where

$$D_q(\alpha e^{i\gamma_\mp}) = \frac{q}{q+1} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^q e^{iq\gamma_\mp}}{\sqrt{q!}}, \tag{12}$$

$$A = i \frac{N(\tau)}{\alpha} e^{-i\tau\left(\omega_2 - \frac{\omega_1}{2} - \frac{\lambda}{2}\right)}, \tag{13}$$

$$\gamma_\mp = \pm \left(\frac{\lambda}{2} \mp \omega_1 \right) \tau, \quad \vartheta = \lambda\tau. \tag{14}$$

Since, in view of Eq. (12), each probability amplitude $D_q(\alpha e^{i\gamma_\mp})$ may be obtained multiplying the q-th poissonian amplitude $e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^q e^{iq\gamma_\mp}}{\sqrt{q!}}$ by $\frac{q}{q+1}$, it is reasonable to guess that, at least in correspondence to not too low values of $|\alpha|^2$, even $D_q(\alpha e^{i\gamma_\mp})$ exhibits a substantial poissonian character.

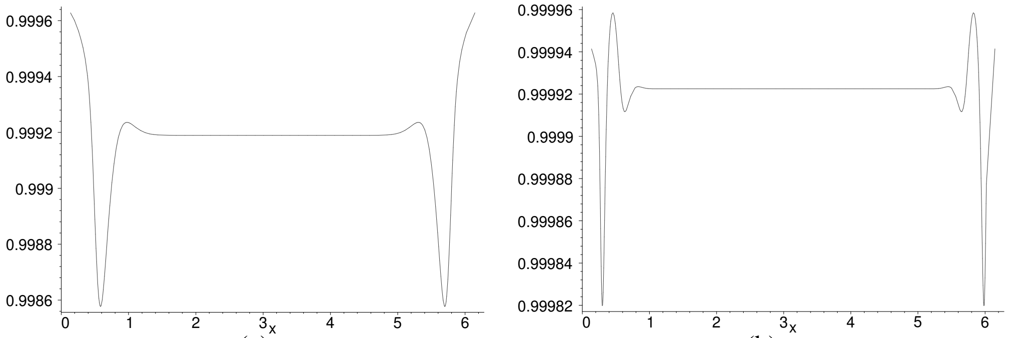
To appreciate this point quantitatively, we take the scalar product between the coherent state $|\alpha e^{i\gamma_\mp}\rangle$ and the normalized state $|d(\alpha e^{i\gamma_\mp})\rangle = d \sum_{q=0}^{\infty} D_q(\alpha e^{i\gamma_\mp})|q\rangle$ obtaining

$$\langle \alpha e^{i\gamma_\mp} | d(\alpha e^{i\gamma_\mp})\rangle = d \left(1 - \frac{1 - e^{-|\alpha|^2}}{|\alpha|^2} \right) \approx d \left(1 - \frac{1}{|\alpha|^2} \right) \tag{15}$$

where

$$d = \left[\sum_{q=0}^{\infty} |D_q(\alpha e^{i\gamma_\mp})|^2 \right]^{-\frac{1}{2}} \simeq 1. \tag{16}$$

Thus for $|\alpha|^2 \geq 10$ we are legitimated to say that at $t = \tau$ the mode of interest is found in a quantum superposition of the two coherent states $|\alpha e^{i\gamma_-}\rangle$ and $|\alpha e^{i\gamma_+}\rangle$.



(a) (b)

Fig. 1. Plot of the function $f(x) = |\langle\varphi_-(x)|\tilde{\varphi}(x)\rangle|$ against $x = \lambda t$ for (a) $|\alpha|^2 = 10$ and (b) $|\alpha|^2 = 20$.

In order to better understand this point, let's consider the normalized state $|\tilde{\varphi}(\tau)\rangle$ defined as

$$|\tilde{\varphi}(\tau)\rangle \equiv \tilde{A}\left[|\alpha e^{i\gamma_-(\tau)}\rangle - e^{-i\vartheta(\tau)}|\alpha e^{i\gamma_+(\tau)}\rangle\right] \tag{17}$$

where the normalization constant $\tilde{A}$ is given by

$$\tilde{A} = \frac{1}{\sqrt{2}}\left[1 - \mathrm{Re}\left(e^{-i\vartheta}e^{-|\alpha|^2\left(1-e^{-i\vartheta}\right)}\right)\right]^{-\frac{1}{2}} \tag{18}$$

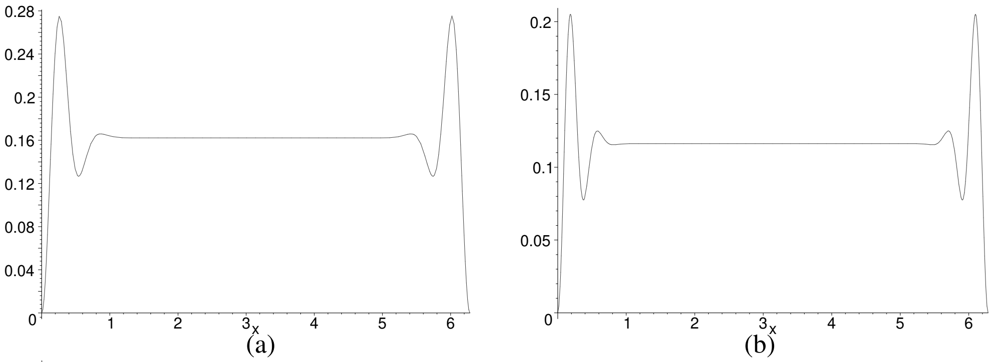
and we write $\vartheta(\tau)$, $\gamma_\mp(\tau)$ instead of ϑ, $\gamma_\mp$.

We have demonstrated that, there exist appropriate conditions, such that the state generated with the help of our scheme is well approximated by the state $|\tilde{\varphi}(\tau)\rangle$ defined in Eq. (17). The results obtained are for simplicity showed in Figs. 1(a)-(b) where we plot the function $f(\tau) = |\langle\varphi_-(\tau)|\tilde{\varphi}(\tau)\rangle|$ in correspondence to $|\alpha|^2 = 10, 20$ respectively.

Looking at these figures it is immediate to deduce that, appropriately choosing the time τ spent by the atom inside the cavity, a measurement of the internal atomic state after the interaction, projects the mode of interest in the desired state if the atom is found in its ground state. This result directly stems from the pecular entanglement which gets established, during the atom-cavity interaction, among the atom, the mode of interest and the auxiliary one.

To claim however that our procedure for generating the linear superposition given by Eq.(17), may be effectively thought of as an experimental scheme, we must, first of all, evaluate its probability of success, that is the probability $P(\tau)$ of detecting the atom in its ground state immediately out of the cavity. This probability depends both on the interaction time τ and on the initial intensity of the mode of interest $|\alpha|^2$. Figures 2(a)-(c) represent $P(\tau)$ in correspondence to $|\alpha|^2 = 10, 15, 20$.

These plots show that, for prefixed values of the initial intensity present in the mode of interest, $P(\tau)$ is practically uniform when τ varies over a wide interval of time. Moreover, we see that $P(\tau)$ may be considered of experimental interest, that is larger than 0.1, when $|\alpha|^2 \leq 20$. This inequality is compatible with the condition on $|\alpha|^2$ under which $|\varphi_-(\tau)\rangle$ may be represented as superposition of two coherent states in accordance with Eq. (17).



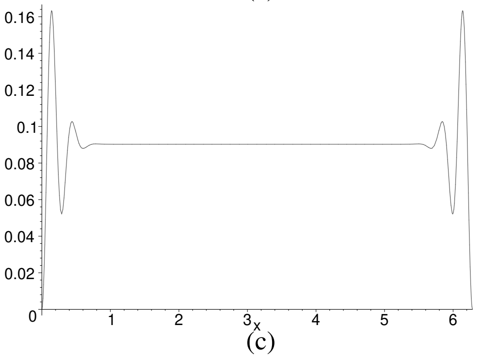


Fig. 2. Probability of success $P(x)$ against $x = \lambda t$ for (a) $|\alpha|^2 = 10$, (b) $|\alpha|^2 = 15$, and (c) $|\alpha|^2 = 20$

Summing up, we wish to remark the main aspects characterizing our scheme. Taking advantage from the peculiar entanglement which gets established during the atom-cavity interaction among the atom, the mode of interest and the auxiliary one, the conditional measurement of the internal state of only one atom immediately after it leaves the resonator, leads to the desired target state. The condition on the initial mean photon number populating the mode of frequency ω_1, that is $10 \leq |\alpha|^2 \leq 20$ is in the actual reach of experimentalists and does not appear as too much restrictive. The free parameter of our conditional measurement is of course the atom-cavity interaction time τ. Controlling the speed of only one atom is easy to realize and, consequently, τ may be tuned to get different desired final states of the mode of interest always in the form of linear superpositions of two coherent states.

In a typical micromaser-based experiment, each atom injected into the resonator flies over a distance which is of the order of a few centimetres with a velocity selected roughly between 10^4cm/sec and 10^7cm/sec [6] with a precision under 2%. Thus, acceptable atom-field interaction times must be chosen between 10μsec and 10 ms. From Figs. 2(a) and 2(c) we estimate τ to be such that $1 \leq \lambda\tau \leq 6$ or, equivalently τ of the order of a few ms.

In the context of our scheme, these values of τ are compatible with the occurrence of a success probability of practical interest and moreover guarantee a remarkable degree of stability of this same probability against unavoidable imprecisions in the control of the atom-field interaction time.

At the end of a successful experiment the specific superposition of coherent states generated in the mode of interest is determined by the effective duration τ. Considering Eq. (14) we immediately deduce that the phase difference $\gamma_- - \gamma_+$ between the two coherent states appearing in Eq. (17), coincides with ϑ whatever τ is. To put better into evidence this peculiar aspect characterizing our scheme, we conveniently re-express Eq. (17) in the form

$$|\varphi_-(\tau)\rangle \approx \tilde{A}\left[|\beta\rangle - e^{-i\vartheta(\tau)}|\beta e^{-i\theta(\tau)}\rangle\right] \tag{19}$$

where $\beta = \alpha e^{i\gamma_-(\tau)}$ and $\theta = \lambda\tau$. Since $|\beta|^2 \equiv |\alpha|^2$ the two coherent components linear superimposed in Eq. (19) belong to the circle singled out by $|\alpha\rangle$ in the phase space relative to the manipulated mode.

It is worth noting that our experimental scheme enables the generation of an even coherent state in correspondence with $\tau = \pi/\lambda$. It is well-known [1] that this state is a nonclassical state. We believe that even the other states producible with the help of the procedure presented in this paper, exhibit nonclassical features which may be traced back to quantum interference between probability amplitudes in the phase space.

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