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DISORIENTED CHIRAL CONDENSATES AND ANOMALOUS PRODUCTION OF PIONS [1]

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The leading-particle effect and the factorization property of the scattering amplitude in the impact parameter space are used to study semiclassical production of pions in the central region. The mechanism is related to the isospin-uniform solution of the nonlinear σ-model coupled to quark degrees of freedom. The multipion exchange potential between two quarks is derived. It is shown that the soft chiral pion bremsstralung also leads to anomalously large fluctuations in the ratio of neutral to charged pion.. We show that only direct production of pions in the form of an isoscalar coherent pulse without isovector pairs can lead to large neutral-charged fluctuations.

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1 Introduction

The old puzzle in cosmic-ray observations is the existence of few events characterized by an anomalously large number of charged pions in comparison with the number of neutral pions, the Centauros [1], indicating that there should exist a strong long-range correlation between two types of the pions. The negative results of the accelerator searches for Centauros at CERN [2,3,4,5] and at Fermilab [6] suggest that the mechanism for their production is not yet well understood and that the production threshold must be larger then 1.8 TeV.

Such long-range correlations are possible if pions are produced semiclassicaly and constrained by global conservation of isospin [7-12].

Although the actual dynamical mechanism of the production of a classical pion field in the course of a high-energy collision is not known, there exist numerous interesting recent theoretical attemps to explain Centauros either as different types of isospin fluctuations due to the formation of a disoriented chiral condensate (DCC) [13-17], or as multiparticle Bose-Einstein correlations (BEC) [18], or as the formation of a strange quark matter (SQM) [19]. Among the most interesting speculations is the idea of DCC that localized regions of misaligned chiral vacuum might

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occur during the ultrahigh-energy hadronic and heavy-ion collisions. These regions, if produced, would behave as a pion laser, relaxing to the ground state by coherent pion emission . It is generally accepted that the fluctuation of the ratio of neutral to charged pions of the Centauro type could be a sign of the DCC formation provided that a single large domain is formed, containing a large number of low p_T pions. Since the pions formed in the DCC are essentially classical they form a quantum superposition of coherent states with different orientation in isospin space. If all the pions in the domain are pointing in the same isospin direction and the condensate state is a pure isoscalar then the probability distribution of the ratio of neutral to total pions in the domain will be given by [10,13,14,16]

$$P_{DCC}(f) = \frac{1}{2\sqrt{f}} \tag{1}$$

where $f = n_0/n$ is the ratio of the number of π_0's in the DCC divided by the total number of pions. The distribution $P_{DCC}(f)$ is clearly very distinct from the binomial distribution which assumes equal probability for production of π_+, π_- and π_0 pion and is of the form

$$P_B(n_0, n) = \binom{n}{n_0} \left(\frac{1}{3}\right)^{n_0} \left(\frac{2}{3}\right)^{n-n_0} \tag{2}$$

which in the limit as $n \to \infty, n_0 \to \infty$ with f fixed, approaches a delta function at $f = 1/3$.

The possibility of observing the DCC type fluctuations critically depends on the size and the energy content of the DCC domain. If this domain is of the pion size, the effect of DCC will be too small to be observed experimentally. Therefore the signals such as the isospin fluctuations, the enhancement of the number of low p_T pions, and the suppresion of HBT correlations are all characteristics of a single large coherent emission domain [20]. The space-time scenario of the formation and decay of the DCC is usually studied within one of the simplified versions of the chiral effective Lagrangians, either the linear or nonlinear sigma model [17,21]. However, the use of sigma models, be they linear or nonlinear, is only a rough approximation to the true situation, because the couplings of pions and sigma to the constituent quarks may be large and their effect should not be ignored.

In the early models [7,8] the coherent production of pions was taken for granted and considered as a dominant mechanism. These models also predict strong negative correlations between the number of neutral and charged pions. In fact, it was observed long ago [9,10] that an uncorrelated jet model involving only pions with the global conservation of isospin gives the same pattern of neutral/charged fluctuations as observed in Centauro events. This strong negative neutral-charged correlation is a general property of the independent pion emission in which the cluster formation (or the short-range correlation between pions) is not taken into account [11,12,22,23]. What makes these older results interesting is the possibility that such a coherent isosinglet pulse of pions could be produced in the high-energy collisions as a result of de-excitation of a highly excited region of space, in which the chiral orientation of the order parameter – the quark condensate – is different from that of the normal vacuum configuration [17,24].

In this paper, following the approach of our earlier papers [23], we consider in Section 2. the leading-particle effect as a source of a classical pion field in the impact parameter space. Since the condition of approximately vanishing isospin of the coherent pulse of pions is crucial for the observation of the DCC phenomena, pions should be produced in the nearly baryon-free central rapidity region .

In Section 3. we show how the factorization property of the scattering amplitude in the impact parameter space may be related to the isospin-uniform solutions of the equation of motion of the nonlinear sigma model in the presence of a classical source containing constituent quarks [25]. We also study in Section 3. the quantum case of the nonlinear σ model and derive the form of the multipion exchange potential between two quarks.

Our results are summarized in the Section 4. Our general conclusion is that within the nonlinear σ model the large isospin fluctuations are consequence of singly produced pions which are constrained by global conservation of isospin.

2 Pion production from a classical source

2.1 The eikonal S matrix with isospin

At high energies most of the pions are produced in the central region. To isolate the central production, we adopt the high-energy longitudinally dominated kinematics, with leading particles retaining a large fraction of their incident momenta. We assume that the collision energy is large enough so that the central region is free of baryons. The energy available for the hadron production is

$$E_{had} = \frac{1}{2}\sqrt{s} - E_{leading} \tag{3}$$

which at fixed total c.m. energy $\sqrt{s}$ varies from event to event. Using the following set of $(3n+2)$ independent variables $s,\ \{\vec{q}_{iT}, y_i\} \equiv q_i,\ \ i = 1, 2, \ldots n$, the n-pion contribution to the s-channel unitarity can be written as an integral over the relative impact parameter b of the two incident leading particles:

$$F_n(s) = \frac{1}{4s} \int d^2b \prod_{i=1}^{n} dq_i \mid T_n(s, \vec{b}; 1 \ldots n) \mid^2, \tag{4}$$

where $dq = d^2q_T dy/2(2\pi)^3$. The normalization is such that

$$\begin{aligned} F_n(s) &= s\sigma_n(s), \\ \sigma_{inel}(s) &= \sum_{n=1}^{\infty} \sigma_n(s). \end{aligned} \tag{5}$$

The basic assumption of the independent pion-emission model, in b-space is the factorization property of the scattering amplitude $T_n(s, \vec{b}; 1 \ldots n)$:

$$T_n(s, \vec{b}; 1 \ldots n) = 2sf(s, \vec{b}) \prod_{\sigma=+,0,-} \prod_{i_\sigma=1}^{n_\sigma} \frac{i^{n_\sigma - 1}}{\sqrt{n_\sigma !}} J_\sigma(s, \vec{b}; q_{i_\sigma}), \tag{6}$$

where, owing to unitarity,

$$\mid f(s, \vec{b}) \mid^2 = e^{-\overline{n}(s, \vec{b})} \tag{7}$$

and

$$\overline{n}(s,\vec{b}) = \sum_{\sigma} \int dq \mid J_{\sigma}(s,\vec{b};q) \mid^2 \tag{8}$$

denotes the average number of emitted pions at a given impact parameter b. The functions $\mid J_{\sigma}(s,\vec{b};q) \mid^2$, after the integration over b, control the shape of the single-particle inclusive distribution. A suitable choice of these functions also guarantee that the energy and the momentum are conserved on the average during the collision. The factorization properties of T_n in the form (6) is a consequence of the pion field satisfying the following equation of motion

$$(\Box + m_{\pi}^2)\vec{\pi}(s,\vec{b};x) = \vec{j}(s,\vec{b};x), \tag{9}$$

where $\vec{j}$ is a classical source related to $\vec{J}(s,\vec{b};q)$ via the Fourier transform

$$\vec{J}(s,\vec{b};q) = \int d^4x e^{iqx}\vec{j}(s,\vec{b};x). \tag{10}$$

The standard solution of Eq.(9) is usually given in terms of in- and out-fields that are connected by the unitary S- matrix $\hat{S}(s,\vec{b};\vec{J})$:

$$\vec{\pi}_{out} = \hat{S}^{\dagger}\vec{\pi}_{in}\hat{S} = \vec{\pi}_{in} + \vec{\pi}_{cl}, \tag{11}$$

where

$$\vec{\pi}_{cl}(x) = \int d^4x' \Delta(x-x';\mu)\vec{j}(s,\vec{b};x'). \tag{12}$$

and $\Delta = \Delta_{ret} - \Delta_{adv}$. The S matrix following from such a classical source is still an operator in the space of pions. The coherent production of pions is described by the following S matrix:

$$\hat{S}(s,\vec{b};\vec{J}) = \int d^2\vec{e} \mid \vec{e}\rangle D(s,\vec{b};\vec{J})\langle\vec{e}\mid, \tag{13}$$

where $\mid \vec{e}\rangle$ represents the isospin-state vector of the leading-particle system. The quantity $D(s,\vec{b};\vec{J})$ is the unitary coherent-state displacement operator defined as

$$D(s,\vec{b};\vec{J}) = exp[\int dq\vec{J}(s,\vec{b};q)\vec{a}^{\dagger}(q) - h.c.], \tag{14}$$

where $\vec{a}^{\dagger}(q)$ denotes the creation operator of the pion field.

2.2 Distribution of pions in isospace

If the isospin of the incoming leading particles is II_3 , then the initial-state vector of the pion field is $\hat{S}(s,\vec{b};\vec{J}) \mid II_3\rangle$, where $\mid II_3\rangle$ denotes a vacuum state with no pions. The n-pion production amplitude is

$$iT_n(s,\vec{b};q_1\ldots q_n) = 2s\langle I'I_3';q_1\ldots q_n \mid \hat{S}(s,\vec{b};\vec{J}) \mid II_3\rangle, \tag{15}$$

where $I'I_3'$ denotes isostate of the outgoing leading particles.

If each possible isospin (I', I_3') of the outgoing leading particle system is produced with equal probability, then we can sum over all $(I'I_3')$ using the group theory alone to obtain the following probability distribution of producing $n_+\pi^+$, $n_-\pi^-$, and $n_0\pi^0$:

$$P_{II_3}(n_+n_-n_0)N_{II_3} = \sum_{I'I_3'} \int d^2b dq_1 dq_2 \dots dq_n \mid \langle I'I_3'n_+n_-n_0 \mid \hat{S}(s, \vec{b}; \vec{J}) \mid II_3 \rangle \mid^2 \quad (16)$$

$$n = n_+ + n_- + n_0 \quad (17)$$

where N_{II_3} is the corresponding normalization factor determined by

$$\sum_{\{n\}} P_{II_3}(n_+, n_-, n_0) = 1.$$

This is our basic relation for calculating various pion-multiplicity distributions, pion multiplicities, and pion correlations between definite charge combinations. In general, the probability $P_{II_3}(n_+n_-n_0)$ would depend on $(I'I_3')$ dynamically. The final-leading-particles tend to favor the $(I', I_3') \approx (I, I_3)$ case. However, if the leading particles are colliding nuclei, an almost equal probability for various (I', I_3') seems reasonable aproximation owing to the large number of possible leading isobars in the final state. In the model leading to Eq.(16) the dynamical information which restricts the possible values $(I'I_3')$ was not taken into account. Nevertheless, even if we assume that $(I', I_3') \approx (I, I_3)$ the pion's cloud could still have the isospin $I_\pi = 0, 2, \dots, 2I$.

Recent studies of heavy-ion collisions at the partonic level [26] argue that the central region is mainly dominated by gluon jets. The valence quarks of the incoming particles which escape from the interaction region form the outgoing leading particle system. Since gluon's isospin is zero, it is very likely that total isospin of the produced pions in the central region is also zero. This picture is certainly true if the central region is free from valence quarks, the situation expected to appear at the extremelly high collision energies.

If the conservation of isospin is a global property of the colliding system, restricted only by the relation

$$\vec{I} = \vec{I'} + \vec{I_\pi}, \quad (18)$$

where $\vec{I_\pi}$ denotes the isospin of the emitted pion cloud, then $\vec{J}(s, \vec{b}; q)$ should be of the form

$$\vec{J}(s, \vec{b}; q) = J(s, \vec{b}; q)\vec{e}, \quad (19)$$

where $\vec{e}$ is a fixed unit vector in isospace independent of q. The global conservation of isospin thus introduces the long-range correlation between the emitted pions.

We shall analyze the isospin structure of the model in the so called grey-disk approximation in which

$$\overline{n}(s, \vec{b}) = \overline{n}(s)\Theta(R(s) - b), \quad (20)$$

where $\overline{n}(s)$ denotes the mean total number of pions produced, $R(s)$ is related to the total inelastic cross section, and Θ is the step function. For $I_3 = I$, it is a straightforward algebra to calculate

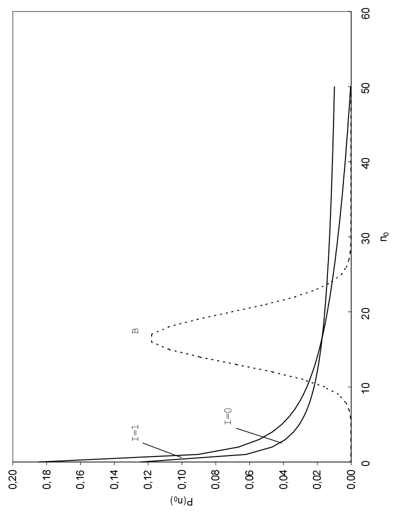

Fig. 1. The distribution of neutral pions as a function of the isospin of the incoming leading-particle system in comparison with the binomial distribution.

the probability of creating n pions of which n_0 are neutral pions:

$$P_I(n_0 \mid n) = \sum_{n_+ + n_- = n - n_0} P_{II}(n_+ n_- n_0) \tag{21}$$

$$= \left(\begin{array}{c} n \\ n_0 \end{array} \right) \frac{B(n_0 + \frac{1}{2}, n - n_0 + I + 1)}{B(\frac{1}{2}, I + 1)}. \tag{22}$$

Here $B(x, y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)}$ is the Euler beta function. Note that $P_I(n_0 \mid n)$ differs considerably from the binomial distribution given here by $3^{-n} 2^{n-n_0} \binom{n}{n_0}$.

In Fig. 1 we show the behavior of P_I for $n = 50$ and isospin $I = 0, 1$ of the initial-leading-particle system in comparison with the corresponding binomial distribution. It is easy to see that in the limit $n \to \infty$, with $f = \frac{n_0}{n}$ fixed, the probability distribution $nP_I(n_0 \mid n)$ scales to the limiting behavior:

$$nP_I(n_0 \mid n) \to P_I(f) = \frac{(1 - f)^I}{B(\frac{1}{2}, I + 1)\sqrt{f}}. \tag{23}$$

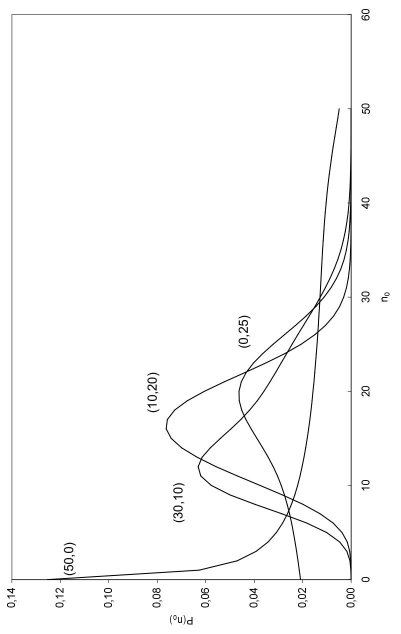

Fig. 2. The curves represent $P(n_0)$ for different combinations of $(\overline{n}_\pi, \overline{n}_\rho)$, the average number of singly produced pions and the average number of ρ-type clusters, respectively.

This limiting probability distribution is different from the usual Gaussian random distribution for which one expects $P_I(f)$ to be peaked at $f = \frac{1}{3}$ as $n \to \infty$. However, if pions are produced through the coherent production of clusters which subsequentlly decay into two or more pions the fraction of neutral pions in an event changes substantially.

2.3 Production of isovector clusters

Let us assume that pions are produced both singly and through isovector clusters of the ρ type[23]. In this case, the most appropriate tool for studying pion correlations is the generating function $G_{II_3}(z, n_-)$:

$$G_{II_3}(z, n_-) = \sum_{n_0, n_+} P_{II_3}(n_+, n_-, n_0) z^{n_0}, \tag{24}$$

from which we can calculate, for example

$$\langle n_0\rangle_{n_-} = \frac{d}{dz}\ln G_{II_3}(1,n_-), \tag{25}$$

$$f^0_{2,n_-} = \frac{d^2}{dz^2}\ln G_{II_3}(1,n_-), \tag{26}$$

and

$$P_{II_3}(n_0) = \frac{1}{n_0!}\frac{d^{n_0}}{dz^{n_0}}\sum_{n_-} G_{II_3}(0,n_-). \tag{27}$$

The form of this generating function in the case of $I = I_3 = 0$ is particularly simple:

$$G_{00}(z,n_-) = \int_0^1 dx \frac{[A(z,x)]^{n_0}}{n_0!} e^{-B(z,x)}, \tag{28}$$

where

$$2A(z,x) = (1-x^2)\overline{n}_\pi + z(1-x^2)\overline{n}_\rho + 2x^2\overline{n}_\rho \tag{29}$$

and

$$2B(z,x) = \overline{n}_\pi(1+x^2-2zx^2) + \overline{n}_\rho(2-z(1-x^2)). \tag{30}$$

Here $\overline{n}_\pi$ denotes the average number of singly produced pions, and $\overline{n}_\rho$ denotes the average number of ρ-type clusters which decay into two short-range correlated pions. Note that $A(1,x) = B(1,x)$.

The total number of emitted pions is

$$\overline{n} = \overline{n}_\pi + 2\overline{n}_\rho. \tag{31}$$

In Fig. 2. we show the behavior of $P(n_0) \equiv P_{00}(n_0)$ for $\overline{n} = 50$ and different combinations of $(\overline{n}_\pi, \overline{n}_\rho)$. We see that Centauro-type behavior is obtained only for $\overline{n}_\pi \neq 0$ and $\overline{n}_\rho = 0$. Recent estimate of the ratio of ρ– mesons to pions, at accelerator energies, is $\overline{n}_\rho = 0.10\overline{n}_\pi$. The behaviour of $\langle n_0\rangle_{n_-}$ and f^0_{2,n_-}can be found in [23].

2.4 Random source

In this section we want to study the multiplicity distribution of pions emitted from a classical random source. In previous sections we have averaged our results only over a chiral orientations of the classical source but not the overall shape of the source.We assume that incident leading particles, which can neither be created nor destroyed, are acting as a classical random source for pions.

The unitary S matrix following from such a classical random source is $\hat{S}(s,\vec{b};\vec{J})$.The initial-state vector for the pion field is $\hat{S}(s,\vec{b};\vec{J}) \mid II_3\rangle$.In practice, we rarely have any information about this initial state.It means that physical quantities should be averaged over the choice of source functions present in the initial state. In quantum statistics, the ensemble average is usually performed using the density operator, which, in our case, is of the form [27]

$$\rho_{II_3}(s,\vec{b}) = [\hat{S}(s,\vec{b};\vec{J}) \mid II_3\rangle\langle II_3 \mid \hat{S}^\dagger(s,\vec{b};\vec{J})]_{av} \tag{32}$$

and normalized to unity: $Tr[\rho_{II_3}] = 1$ In terms of the pion-number operator

$$\begin{aligned} N &= N_+ + N_- + N_0 \\ &= \sum_\sigma \int dq a^\dagger_\sigma(q) a_\sigma(q) \end{aligned} \tag{33}$$

the pion-generating function in the impact-parameter space becomes

$$G_{II_3}(z_0, z_+, z_-) = Tr[\rho_{II_3}(s,\vec{b}) : e^{\sum_\sigma (z_\sigma - 1) N_\sigma} :] \tag{34}$$

$$= \langle exp[\sum_\sigma (z_\sigma - 1)\overline{n}_\sigma(s,\vec{b})] \rangle_{II_3} \tag{35}$$

where

$$\overline{n}_\sigma(s,\vec{b}) = \int dq \mid J_\sigma(s,\vec{b};q) \mid^2 . \tag{36}$$

The global conservation of isospin and the identity of pions require

$$J_\sigma(s,\vec{b};q) = J(s,\vec{b};q)\vec{e_\sigma} \tag{37}$$

where

$$e_0 = e_3 \tag{38}$$

$$e_\pm = \frac{1}{\sqrt{2}}(e_1 \mp i e_2). \tag{39}$$

Since

$$\overline{n}_\sigma(s,\vec{b}) = \mid e_\sigma \mid^2 \overline{n}(s,\vec{b}) \tag{40}$$

we can find the generating function in b-space for producing n_{ch} charged and n_0 neutral pions which is of the form

$$G_{II_3}(z_0, z_{ch}) = (I + \frac{1}{2}) \frac{(I - I_3)!}{(I + I_3)!} \int_{-1}^{1} dx \mid P_I^{I_3}(x) \mid^2 \langle e^{\overline{n}(s,\vec{b}) z(x)} \rangle \tag{41}$$

where

$$z(x) = x^2 z_0 + (1 - x^2) z_{ch} - 1 \tag{42}$$

and $P_I^{I_3}$ is the associate Legendre polynomial. The case $I = I_3 = 0$ is particularly simple. The corresponding generating function is

$$G(z_0, z_{ch}) = \int_0^1 \frac{df}{2\sqrt{2}} \langle e^{\overline{n} z(f)} \rangle \tag{43}$$

where $G \equiv G_{00}$ and

$$z(f) = f z_0 + (1 - f) z_{ch} - 1. \tag{44}$$

For simplicity of writing we have left out the $(s, \vec{b})$ dependence in $\overline{n}(s, \vec{b})$.

If the classical random source is such that its fluctuations about the mean are Gaussian[27], the generating function becomes

$$G(z_0, z_{ch}) = \int_0^1 \frac{df}{2\sqrt{2}} e^{\langle \overline{n} \rangle z(f) + \frac{1}{2} d^2 z^2(f)} \tag{45}$$

where

$$d^2 = \langle \overline{n}^2(s, \vec{b}) \rangle - \langle \overline{n}(s, \vec{b}) \rangle^2. \tag{46}$$

The form of this generating function is such that the Centauro type behaviour of $P(n_0)$ can be expected only in the limit of a very small dispersion d in the number of produced pions in b-space. This problem certainly requires further study, in particular, if we go beyond the Gaussian approximation for the fluctuations of the source about the mean.

3 Relation to the nonlinear sigma model

In this section we study the relationship that can be established between the nonlinear sigma model coupled to quarks and our eikonal model with factorization.

3.1 Quark sources of the pion field

As is well known, the Lagrangian for QCD with two light up and down quarks has an approximate global $SU(2)_L \times SU(2)_R$ symmetry, which at low temperatures, is spontaneously broken to $SU(2)_V$ by a nonzero value of the quark condensate $\langle \bar{q}_L q_R \rangle$, which is regarded as an order parameter of the system. This order parameter can be represented as a four-component vector $\phi \equiv (\sigma, \vec{\pi})$ buildt from the quark densities.The chiral symmetry then corresponds to $O(4)$ rotations in internal space.

The true vacuum of the theory is defined as $\langle \phi \rangle = (\langle \sigma \rangle, \vec{0})$, with $\langle \sigma \rangle \neq 0$. In QCD the spontaneous symmetry breakdown leads to nearly massless Goldstone bosons (the pions) and gives the constituent-quark mass. At low energies and large distances (momentum scale smaller than $1 GeV$) the dynamics of QCD is described by an effective Lagrangian containing the $\sigma, \vec{\pi}$ fields and constituent quarks. Recently, the idea that extended regions of the misaligned chiral field ϕ may be formed in a very high energy hadron-hadron, hadron-nucleus or nucleus-nucleus collisions, has attracted a lot of interest both theoretically and experimentally [17].If a single large DCC domain forms, its decay into pions can lead to anomalous fluctuations in the ratio of neutral to charged pions similar to ones observed in Centauro-type events.

In the DCC dynamics we distinguish three stages: formation, evolution and decay stage. In the conventional approach [16] one starts with a chirally symmetric phase at $T > T_c$ and DCC formation happens as T, due to a rapid expansion or cooling, drops below T_c spontaneously breaking the chiral symmetry.

The evolutionary stage of the DCC is usually described by the classical chiral dynamics based on the σ-model, mostly the linear σ- model. For the purpose of comparison with our eikonal model we consider the nonlinear σ-model coupled to quarks at zero temperature [25] which is expected to describe the late stage of the DCC evolution.

The Lagrangian for the nonlinear σ-model coupled to quarks is

$$L = \frac{f_\pi^2}{4} Tr(\partial_\mu U^\dagger \partial^\mu U) + \overline{q}(i\gamma\partial)q - gf_\pi \overline{q}Uq \tag{47}$$

where

$$U = exp(i\gamma_5 \frac{\vec{\pi}\cdot\vec{\tau}}{f_\pi}) \tag{48}$$

We parametrize the pion field in the following form

$$\vec{\pi}(x) = f_\pi \vec{n}(x)\theta(x) \tag{49}$$

where $\vec{n}(x)$ is an unit vector which determines the isospin orientation of the pion field, obeying $\vec{n}^2 = 1$.

The Euler-Lagrange equations of motion for θ and $\vec{n}$ are

$$\begin{aligned} \Box\theta - \sin\theta\cos\theta\partial_\mu\vec{n}\cdot\partial^\mu\vec{n} &= -i\frac{m_Q}{f_\pi^2}\vec{n}\cdot(\bar{Q}\vec{\tau}\gamma_5 Q) \\ \partial_\mu(\sin^2\theta\vec{n}\times\partial^\mu\vec{n}) &= -i\frac{m_Q}{f_\pi^2}\vec{n}\times(\bar{Q}\vec{\tau}\gamma_5 Q)\sin\theta \qquad (50) \\ & \qquad (51) \end{aligned}$$

where Q denotes the constituent quark defined by

$$Q = e^{i\gamma_5 \frac{\vec{\pi}\cdot\vec{\tau}}{2f_\pi}} q \tag{52}$$

and $m_Q = gf_\pi$ is the constituent quark mass. We treat

$$-i\frac{m_Q}{f_\pi}\bar{Q}\vec{\tau}\gamma_5 Q = \vec{j}(x) \tag{53}$$

as a given classical external source and consider the class of solutions that can be rotated into a uniform one, $\vec{n}(x) =$ constant, known as the Anselm-class of solutions [13]. The solutions with constant $\vec{n}$ can be realized if the source points to a certain fixed direction $\vec{e}$ in the isospace:

$$\vec{j}(x) = j(x)\vec{e} \tag{54}$$

The source term in the equation of motion for $\vec{n}$ disappears if we choose

$$\vec{n}(x) = \vec{e}. \tag{55}$$

Then the equation of motion for the pion field reduces to

$$\Box\theta(x) = j(x) \tag{56}$$

with

$$\vec{\pi}(x) = f_\pi\theta(x)\vec{e} \tag{57}$$

We see that this is exactly the equation of motion for the pion field that we have used in our eikonal model in order to predict the appearance of the Centauro type behaviour in heavy-ion and hadron collisions. This relationship therefore offers the possibility for studying the importance of various quark sources responsible for the DCC formation.

3.2 Quantum nonlinear sigma model

In quantum chiral field theory of the nonlinear σ-model the role of a strong coupling of $q\pi$-interaction has the quantity m_Q/f_π. However, in phenomenological analyses the strong coupling constant of $q\pi$-interaction is g_Q. It is connected with m_Q/f_π through the Goldberger-Treiman relation

$$g_Q = g_A \frac{m_Q}{f_\pi} \tag{58}$$

where g_A denotes the axial vector current constant whose value, if different from unity, should come, according to Lehmann [28], from higher orders of perturbation theory. Since the Lagrangian of the nonlinear σ-model is nonpolynomial, a suitable renormalization procedure and operator normal ordering should be formulated [29]. In this respect, as a first step, we derive the form of the multipion propagator between two quarks. Let

$$\Phi =: e^{i\gamma_5 \frac{\vec{\pi}\vec{\tau}}{f_\pi}} - 1 : \tag{59}$$

denote the chiral super field of the pion. The Lagrangian describing the interaction of this super chiral field with quarks is

$$L = \frac{f_\pi^2}{4} Tr(\partial_\mu \Phi \partial^\mu \Phi) + \bar{q}(i\gamma\partial - m_Q)q - \frac{m_Q}{f_\pi}\bar{q}\Phi q. \tag{60}$$

The chiral super propagator of the field Φ is defined by

$$\triangle_\Phi(x) = \langle T(\Phi(x)\Phi(0))\rangle \tag{61}$$

Its form, after number of algebraic manipulations, is

$$\triangle_\Phi(x) = \{1 \otimes 1 + \frac{f_\pi^2}{3}(\gamma_5\vec{\tau}) \otimes (\gamma_5\vec{\tau})\partial_\triangle\}\partial_\triangle[\triangle ch(\frac{\triangle}{f_\pi^2})] + 1 \tag{62}$$

where

$$\langle T(\pi_i(x)\pi_j(0))\rangle = \delta_{ij}\triangle(x). \tag{63}$$

and

$$\triangle(x) = \frac{1}{4\pi^2}\frac{1}{x^2 - i\epsilon}$$

.

The multipion exchange potential between two quarks is related to the Fourier transform of $\triangle_\Phi(x)$ in the following way:

$$\bar{u}(p_1')\bar{u}(p_2')\tilde{\triangle}_\Phi(q)u(p_1)u(p_2) = \omega_1'\omega_2'\tilde{V}_\Phi(q)\omega_1\omega_2 \tag{64}$$

where

$$u(p) = \sqrt{\frac{\epsilon_p + m}{\epsilon_p}}\begin{pmatrix}\omega \\ \omega'\end{pmatrix}$$

and

$$\omega' = \frac{\vec{\sigma}\vec{p}}{\epsilon_p + m}\omega$$

with $\omega^*\omega = 1$.

We should also mentioned that the soft chiral pion bremsstralung [30-33], in which every incoming and outgoing quark line is replaced by

$$q \longrightarrow \exp(i\gamma_5 \frac{\vec{\pi}\vec{\tau}}{2f_\pi})q \tag{65}$$

also leads to the distribution of neutral pion of the form $1/\sqrt{n_0 n}$ which is typical for coherent pion production without invoking the notion of DCC formation. We conclude therefore that $nP(n_0 \mid n) \sim \sqrt{n/n_0}$ is not always a definite signature of DCC formation.

4 Conclusion

The result of our analysis are the following:

- Within the framework of an unitary eikonal model with factorization and global conservation of isospin the Centauro-type behaviour can only be expected for pions which are produced singly (see Fig.1 and 2).
- The Centauro-type effect depends on isospin of the initial-leading- particle system (see Fig.1).
- The coherent production of ρ-type clusters supresses the Centauro-type behaviour (see Fig. 2).
- A classical random pion source, which has Gaussian fluctuations about the mean, also supresses the Centauro-type behaviour.
- The leading-particle effect and the factorization property of the scattering amplitude in the impact parameter space may be related to the isospin-uniform solutions of the nonlinear σ-model coupled to quarks.
- The multipion exchange potential between two quarks is derived which is different from the one-gluon exchange potential.
- The soft chiral pion bremsstralung also leads to anomalously large Centauro-type ratio of neutral to charged pions.

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