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ANOMALOUSLY LARGE C-AXIS CRITICAL CURRENT DENSITY IN Bi2212 THIN-FILM STACKS: INTERPRETATION IN TERMS OF SUPERCONDUCTING FILAMENTS

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In Bi2212 thin-film based stacks, c-axis critical currents larger than the depairing critical current have been observed. It has been proposed recently that flux lines bound to screw dislocations with Burgers vectors parallel to the c axis of layered superconductors can lead to such an enhancement of the c-axis critical current, if the superconducting samples are small with respect to the c-axis penetration depth. We provide further arguments in favour of this interpretation.

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1 Introduction

The cuprate superconductors consist of ab-plane superconducting layers weakly coupled in the perpendicular (c-axis) direction [1]. Thus, the current density describing the flow between two superconducting layers with a gauge-invariant phase difference $\delta\theta_{\rm g.i.}$ is

$$j = j_c \sin \delta\theta_{\rm g.i.}, \tag{1}$$

where j_c is the depairing critical current density in the c-axis direction.

The layered structure of the cuprates results in a large ab plane *vs.* c axis anisotropy of the electron motion (for a review, see [2]). In particular, the penetration depth for currents flowing in the ab plane is much shorter than for currents along the c axis, $\lambda_{ab} \ll \lambda$. There is a simple relation between j_c and λ. In fact, within linear response theory $\delta\theta_{\rm g.i.} \ll 1$ and we can therefore write Eq. (1) in the form of the London equation,

$$j = -\frac{1}{\mu_0 \lambda^2}\left(A_z - \frac{\Phi_0}{2\pi}\frac{\partial\theta}{\partial z}\right), \tag{2}$$

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where Φ_0 is the superconducting flux quantum. By comparing Eqs. (1), (2), the depairing critical current density along the c axis can be written

$$j_c = \frac{\Phi_0}{2\pi\mu_0\lambda^2 d}, \tag{3}$$

where d is the distance between the neighbouring superconducting layers.

Because of its implications for the interlayer pairing theory of Anderson, the c axis penetration depth λ has received much experimental attention recently. There is consensus that λ is a strongly doping-dependent quantity which increases with decreasing the number of holes in the CuO_2 planes. For $Bi_2Sr_2CaCuO_2$ (Bi2212) which we discuss here, there is some scatter in the absolute values of λ. For instance, according to the recent measurements of Gaifullin *et al.* and Shovkun *et al.*, $\lambda \approx 300$ μm for underdoped samples with $T_c = 68$ K [3], 150 μm for optimally doped samples [3, 4], and 50 μm for overdoped samples with $T_c = 84$ K [4], whereas an older paper by Jacobs *et al.* reports $\lambda \approx 40$ μm for an optimally doped sample [5]. We take $\lambda \approx 40$ μm and $d \approx 15$ A , and obtain an upper bound on the c axis depairing critical current in Bi2212 at all dopings $j_c < 1.1 \times 10^4$ A/cm^2. This upper bound is consistent with direct critical current measurements. In fact, for single crystals, $j_c < 700$ A/cm^2 has been reported for underdoped and optimally doped samples [6] (see also [1]), whereas for overdoped crystals $j_c < 7.2 \times 10^3$ A/cm^2 [1] has been found. For carefully prepared thin films, Inoue *et al.* have reported j_c between 10 A/cm^2 (for underdoped films) and 1×10^4 A/cm^2 (for overdoped films) [7]. It is worth pointing out that the critical current densities measured in [7] are in a very good agreement with Eq. (3) and the c-axis penetration depths reported in [3, 4].

The work reported here is motivated by the experimental study of Xiao *et al.*, who have observed c-axis critical currents in Bi2212 thin films as large as 2×10^4 A/cm^2 [8], well above our estimated upper bound on the depairing critical current. Usually, the maximal dissipationless current (critical current) which can be passed through a type-II superconductor is much smaller than the depairing current, and is determined by the complex physics of vortices and pinning. In our opinion, the key to solving this paradox is that the ab-plane dimensions of the samples studied by Xiao *et al.* were smaller than λ.

In this paper we further elaborate on the recently proposed interpretation [9] of the experiment of Xiao *et al.* in terms of superconducting filaments in small samples. The plan of the paper is as follows. In Section 2 we explain why flux lines bound to screw dislocations can be viewed upon as superconducting filaments. In Section 3 we discuss the interaction between the screw dislocation and the flux line and show that it is, contrary to the generic case in low-T_c superconductors, attractive. In Section 4 we study whether, even for large cross-section areas, the dislocations close to the ab-plane circumference of the superconducting wire can lead (due to a possibly imperfect screening) to a finite contribution to the c-axis supercurrent.

2 Superconducting filaments

Xiao *et al.* have measured the c-axis critical currents on thin-film based stacks depicted schematically in Fig. 1. The height h of the stacks was 80-200 nm, while their cross-section area was between 2×2 μm^2 and 50×50 μm^2. The films were nearly perfectly epitaxial, as indicated by the transmission electron microscopy and rocking curve measurements.

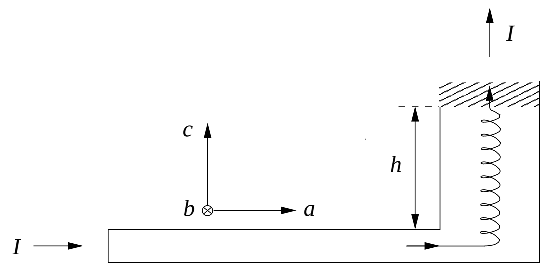

Fig. 1. The thin-film based stack studied by Xiao *et al.* [8]. The current I is fed into the c-axis oriented thin film and is collected in a gold contact (shaded area) on top of the stack with height h. A superconducting filament straddling the stack is shown schematically.

We believe the observed high critical current densities were due to linear paths of strong superconductivity straddling the entire height of the stacks, which we call superconducting filaments. In order to justify this hypothesis, we have to show that (i) in the thin films, there exist linear defects along which such filaments could form and (ii) the linear defects enable the formation of the superconducting filaments.

(i) In any crystal at a finite temperature, there exist natural linear defects: dislocations. In particular, it has been shown some time ago that screw dislocations are quite common in cuprate thin films [10].

(ii) If a flux quantum is spontaneously attached to a screw dislocation whose Burgers vector is parallel to the c axis, then the same current I_0 which circulates around the center of the flux quantum adds also to the c-axis supercurrent, because the current distribution resembles that of a solenoid with radius $\sim \lambda_{ab}$. In a macroscopic description averaging over distances larger than λ_{ab}, the current density due to the bound state of the dislocation and the flux quantum is $\mathbf{j} = (0, 0, j)$ with $j = I_0 \delta^2(\mathbf{r} - \mathbf{r}_0)$, where $\mathbf{r}_0$ is the in-plane coordinate of the vortex and

$$I_0 = \frac{\Phi_0 d}{2\pi \mu_0 \lambda_{ab}^2} K_0 \left(\frac{\xi_{ab}}{\lambda_{ab}} \right) . \tag{4}$$

Here d is the c-axis lattice constant of the superconductor, ξ_{ab} is the in-plane coherence length, Φ_0 is the superconducting flux quantum, and K_0 is the zero-order Macdonald function. Thus, a superconducting filament is formed.

Finally, we come to the role played by the cross section of the stack. In a stack with an infinite cross section, the current I_0 carried by the filament in the c-axis direction is completely screened by a current $-I_0$ along the c axis [11], in order that far away from the dislocation the magnetic field $\mathbf{B} = 0$, as it should be in a superconductor. However, since this latter screening current is distributed on the length scale $\sim \lambda$, it is clear that for stacks with ab-plane dimensions much less than λ the screening current cannot develop and the filament does contribute to the c-axis supercurrent.

3 Pinning by the screw dislocation

In order to show that the filament picture applies to the experiment of Xiao *et al.*, we have to make sure that the screw dislocations do not repel the flux lines. In our original paper [9], we have considered two contributions to the pinning force due to screw dislocations, both of which are attractive: (i) an interaction of purely electromagnetic origin discovered by Ivlev and Thompson [12], and (ii) the core pinning interaction (see [13] for a classification of pinning forces), which should be strong due to the d-wave symmetry of pairing in the cuprates.

However, there exist another contributions to the interaction between dislocations and flux lines, which are due to the mutual influence of non-zero strains around the dislocations and of the non-vanishing difference of elastic properties in the normal and superconducting states [13]. Such interactions are usually repulsive in conventional superconductors [14] and have not been included in our original paper.

In general, there exist two mechanisms of pinning by the elastic energy: "the volume effect" and "the second-order interaction" [13]. The volume effect comes from basic thermodynamics: the volume change δV of an element of a superconductor between the normal and superconducting states in the presence of a finite pressure p due to the dislocation results in the change of energy $-p\delta V$. However, since the stress field of the screw dislocation is pure shear, there is no volume effect in our case [13].

The second-order interaction is due to the change in elastic constants. The strain field at a distance r from the screw dislocation is $\varepsilon = d/\pi r$ and the elastic energy per unit length of the dislocation is (assuming tetragonal symmetry)

$$E_{\rm elast} = \frac{1}{2}\int d^2\mathbf{r} C_{44}\varepsilon^2. \tag{5}$$

Note that the elastic modulus C_{44} is a function of $\mathbf{r}$, and the form of $C_{44}(\mathbf{r})$ depends on the position of the flux line. The difference of the elastic energy for a flux line coinciding with the dislocation and far apart from it, $\delta E_{\rm elast}$, defines the interaction energy of the flux line and dislocation. A simple estimate using Eq. (5) yields

$$\delta E_{\rm elast} \approx \left(C_{44}^N - C_{44}^S\right)(d/\pi)^2 \ln(\xi/a),$$

where a is a cut-off of the order of the in-plane lattice constant, and C_{44}^N, C_{44}^S are the elastic moduli in the normal and superconducting states, respectively. In simple elemental superconductors such as Nb, Pb, and V, the elastic moduli decrease when the sample is cooled to the superconducting state [15]. Thus $\delta E_{\rm elast} > 0$ and the dislocations repel the flux lines.

Unfortunately, we are not aware of a detailed study of the elastic moduli in Bi2212. But in $La_{2-x}Sr_xCuO_4$, the situation seems to be reverted with respect to the elemental superconductors and most of the elastic moduli increase in the superconducting state [16]. The C_{44} modulus, on the other hand, does not change at T_c within the resolution of the experiment [16]. If we assume that Bi2212 behaves in a similar way, we are led to conclude that pinning to c-axis screw dislocations by the elastic energy is negligible in the cuprates.

Finally, a recent study by Dam *et al.* [17] seems to provide a direct experimental confirmation of the attractive total interaction between the flux lines and c-axis screw dislocations. In fact, Dam *et al.* have found that the in-plane critical current of $YBa_2Cu_3O_7$ thin films is independent of the transverse magnetic field B for $B < B^\star$, where the characteristic field $B^\star \approx 0.7 n_{\rm disl}\Phi_0$, and $n_{\rm disl}$ is the dislocation density. This demonstrates that nearly every dislocation is able to pin a flux line. Moreover, the pinning must be by attraction, since for a repulsive interaction, the critical current should decrease with increasing B even for $B < B^\star$.

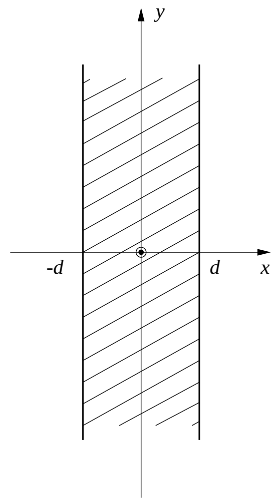

Fig. 2. Sketch of the physical system that is being modelled. A superconducting slab is localized between two semi-infinite regions occupied by the vacuum. An infinitely thin electrically isolated wire carries the current I_0 along the positive z axis.

4 Filaments close to the superconductor surface

In [9] it was shown that, for superconductors whose cross section is a half-plane, the screening current exactly cancels the current carried by the filament, irrespective of the distance of the screw dislocation from the planar surface of the superconductor. In this Section we discuss the question about the influence of the superconductor surface on the screening current in more detail. Solving for the current distribution around a screw-dislocation source in a superconducting slab, we show that perfect screening does take place even if one of the dimensions of the cross section is much smaller than λ, provided the other dimension of the cross section is sufficiently large (infinite in our example). In addition, we study the asymptotic behaviour of the screening current at large distances from the screw dislocation. We find that, unlike in the bulk where the screening current decays exponentially at large distances from the source, in the presence of a surface the screening current exhibits only a power law decay.

The physical system to be investigated is illustrated in Fig. 2. In a Cartesian frame of reference (x, y, z), the superconducting slab extends from $x = -d$ to $x = d$, and is infinitely elongated in both the y and z directions. The regions $x < -d$ and $x > d$ are occupied by the vacuum. We model the flux line bound to the screw dislocation by an infinitely thin electrically isolated wire carrying the current I_0 along the (positive) z axis. Our aim is to find the supercurrent $\mathbf{j}$ which screens the magnetic field $\mathbf{B}$ generated by the wire.

Let us introduce a vector potential $\mathbf{A}$ such that $\mathbf{B} = \nabla \times \mathbf{A}$. The symmetry of the problem allows us to assume $\mathbf{A} = (0, 0, A)$, where $A = A(x, y)$. Note that in our gauge $\nabla \cdot \mathbf{A} = 0$. Making use of Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ and of the London equation $j = -A/\mu_0\lambda^2$, the equations for A in the superconducting slab ($-d < x < d$) and in the vacuum ($x < -d$ or

$x > d$) are found to be, respectively,

$$(\lambda^{-2} - \nabla^2)A = \mu_0 I_0 \delta(x)\delta(y), \quad (6)$$

$$\nabla^2 A = 0. \quad (7)$$

Since we are searching only for even solutions $A(x, y) = A(-x, y)$, we can restrict our study to the region $x > 0$ and we have to require that $\partial A/\partial x|_{x=0}$ vanishes. The system of Eqs. (6), (7) is closed with the boundary conditions requiring the continuity of A and $\partial A/\partial x$ across the boundary $x = d$. We enforce also $A \to 0$ as $x \to \infty$ and $y \to \pm\infty$.

Eq. (7) is autonomous in x and y, and the corresponding domain of interest is unbounded in y and in the positive direction of x. Therefore, the general solution to Eq. (7) is separable and can be expressed as

$$A(x, y) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk a_k \cos(ky) \mathrm{e}^{-k(x-d)}. \quad (8)$$

Eq. (6) is also autonomous in x and y. However, the domain is unbounded in y only, and hence the general solution for $-d < x < d$ can be written as

$$A(x, y) = A_{\mathrm{sing}}(x, y) + A_{\mathrm{reg}}(x, y) \quad (9)$$

with

$$A_{\mathrm{sing}}(x, y) = \frac{\mu_0 I_0}{2\pi} K_0 \left(\frac{\sqrt{x^2 + y^2}}{\lambda} \right), \quad (10)$$

$$A_{\mathrm{reg}}(x, y) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk b_k \cos(ky) \cosh(x\sqrt{k^2 + \lambda^{-2}}). \quad (11)$$

Note that the singular part of Eq. (9), A_{sing}, represents a particular solution to the inhomogeneous Eq. (6). Let us point out that Eq. (9) is (by construction) even in x, as it should be.

Next, let us define the Fourier expansion coefficients c_k and d_k through the following expressions

$$A_{\mathrm{sing}}(d, y) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk c_k \cos(ky),$$

$$\frac{\partial A_{\mathrm{sing}}}{\partial x}(d, y) = \sqrt{\frac{2}{\pi}} \int_0^\infty dk d_k \cos(ky). \quad (12)$$

Then, on matching the solutions Eqs. (8), (9) at $x = d$ we find

$$a_k = \frac{c_k\sqrt{k^2 + \lambda^{-2}} \sinh(\sqrt{k^2 + \lambda^{-2}}d) - d_k \cosh(\sqrt{k^2 + \lambda^{-2}}d)}{\sqrt{k^2 + \lambda^{-2}} \sinh(\sqrt{k^2 + \lambda^{-2}}d) + k \cosh(\sqrt{k^2 + \lambda^{-2}}d)},$$

$$b_k = \frac{-d_k - k c_k}{\sqrt{k^2 + \lambda^{-2}} \sinh(\sqrt{k^2 + \lambda^{-2}}d) + k \cosh(\sqrt{k^2 + \lambda^{-2}}d)}. \quad (13)$$

Eqs. (8), (9), (13) together with the inverse of Eqs. (12) enable us to construct the solution $A(x, y)$. In Fig. 3 we show the contour plot of $A(x, y)$ inside the superconducting slab for the case $\alpha = 1$, where $\alpha = d/\lambda$.

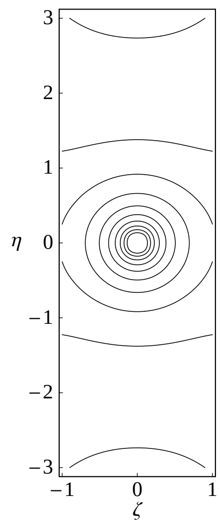

Fig. 3. Lines of constant vector potential A around an electrically isolated wire inside a superconducting slab for $\alpha = 1$. Here, $\zeta = x/\lambda$ and $\eta = y/\lambda$ are the dimensionless coordinates. The contour values range from $A/\mu_0 I_0 \approx 0.017$ to $A/\mu_0 I_0 \approx 0.349$ with ≈ 0.037 increments.

Now we can discuss the main quantity of interest, namely the total screening current, I, inside the superconducting slab. I can be obtained by integrating the London equation over the cross section of the slab:

$$I = -\frac{1}{\mu_0 \lambda^2} \int_{-d}^{d} dx \int_{-\infty}^{\infty} dy A(x, y). \tag{14}$$

The singular part of Eq. (9), A_{sing}, contributes $I_{\text{sing}} = -I_0 + \delta I$, where

$$\delta I = \frac{2\alpha I_0}{\pi} \int_0^{\infty} \frac{dt}{\sqrt{\alpha^2 + t^2}} K_1 \left(\sqrt{\alpha^2 + t^2} \right) \tag{15}$$

and K_1 is the Macdonald function of the first order. The regular part, A_{reg}, contributes $I_{\text{reg}} = -\delta I$, so that the total screening current $I = I_{\text{reg}} + I_{\text{sing}} = -I_0$. Note that this conclusion is independent of α. Thus there is perfect screening even for thin slabs with $d \ll \lambda$.

There is an important question of how fast does the screening current saturate (with increasing y dimension of the slab) to $-I_0$. In a thick slab $d \gg \lambda$, $A \approx A_{\text{sing}}$ and the answer is clear: the screening takes place at distances $\sim \lambda$ from the screw dislocation, and at larger distances A is exponentially small. A more interesting and perhaps unexpected result emerges when we focus on superconducting slabs which are not in the thick limit, for which A_{reg} can not be neglected. In that case we find that at large $|y|$, $A \approx A_{\text{reg}} \propto y^{-2}$ and the screening current decays much more slowly than for thick slabs. In fact, restricting ourselves for simplicity to the plane $x = 0$ and taking $\alpha = 1$, we find that for $|\eta| \gg 1$ (where $\eta = y/\lambda$)

$$\frac{A_{\text{reg}}(0, y)}{\mu_0 I_0} \sim 0.115234\eta^{-2} + 0.407233\eta^{-4} + 0.081128\eta^{-6}. \tag{16}$$

The general procedure that we have used to obtain the asymptotic expansion Eq. (16) is presented in the Appendix. Applying the same procedure to the case $x = d$ and taking again $\alpha = 1$, we obtain the leading asymptotic behaviour $A_{\rm reg}(d,y)/\mu_0 I_0 \sim 0.177822\eta^{-2}$. Note that along both $x = 0$ and $x = d$, we find $A_{\rm reg} \propto y^{-2}$.

5 Conclusions

In conclusion, we have provided further arguments supporting our previous interpretation of the observed anomalous c-axis critical current densities in Bi2212 thin films [8] in terms of superconducting filaments [9]. In particular, we have argued that the interaction between the c-axis screw dislocations and flux lines is attractive and thus the superconducting filaments are stable. Moreover, within a model calculation we have shown that the bulk current which screens the filament decays only as a power-law of the distance from the filament. Thus, even stacks with cross-section dimensions comparable to the c-axis penetration depth λ should exhibit enhanced critical current densities, if they contain c-axis screw dislocations.

An alternative explanation of the large c-axis critical currents in Bi2212 thin films and of the associated absence of intrinsic Josephson phenomena has been given by Inoue *et al.* [7]. Similarly to the filament picture, Inoue *et al.* attribute these effects to imperfections of the thin films. They differ, however, in their choice of the relevant imperfections: in [7] it is assumed that the strong c-axis superconductivity which dominates the c-axis transport is realized at the boundaries of the grains forming the film.

By applying a high-temperature annealing in O_2, Inoue *et al.* were able to grow high-quality thin films with grains larger than 5 μm in diameter, which did exhibit intrinsic Josephson phenomena [7]. This was taken as an evidence that it is the grain boundaries which short-circuit the c-axis supercurrent. However, the high-temperature annealing presumably lowers also the screw dislocation density. Therefore the results of [7] might be consistent also with the filament picture. Further measurements are needed in order to discriminate between these two scenaria.

Appendix

We wish to establish the asymptotic behaviour of $A_{\rm reg}$ in the superconducting slab for $x = 0$ and large y. The approximate technique we shall use is based on the Riemann-Lebesgue lemma [18] which follows.

Consider a Fourier-type integral of the form

$$F(\eta) = \int_0^\infty dk e^{ik\eta} q(k), \tag{17}$$

where the function $q(k)$ does not depend on the positive parameter η. Let $q(k)$ be continuously differentiable in the interval $[0,\infty)$, $q^{(s)}(k)$ be its s-th derivative, and the definite integrals $\int_0^\infty dk e^{ik\eta} q^{(s)}(k)$, $s = 1, 2, \ldots$ are uniformly convergent for all sufficiently large values of η. Then the expression

$$F(\eta) \sim \sum_{s=0}^{\infty} q^{(s)}(0) \left(\frac{i}{\eta}\right)^{s+1}, \quad \eta \to \infty \tag{18}$$

is an asymptotic expansion of Eq. (17). We only note that the way to obtain Eq. (18) is via integrating by parts Eq. (17), yielding successively new terms of the expansion.

For the purpose of the present problem, we introduce $F(\eta, t)$ by writing

$$A_{\rm reg}(0,\eta) = \frac{\mu_0 I_0}{\pi^2} {\rm Re} \int_0^\infty dt F(\eta, t), \tag{19}$$

where $F(\eta, t) = \int_0^\infty dk e^{ik\eta} q(k,t)$,

$$\begin{aligned} q(k,t) &= \frac{\cos(kt)}{\sqrt{k^2+1}\sinh(\alpha\sqrt{k^2+1}) + k\cosh(\alpha\sqrt{k^2+1})} \\ &\times \left[\frac{\alpha}{\sqrt{\alpha^2+t^2}} K_1(\sqrt{\alpha^2+t^2}) - kK_0(\sqrt{\alpha^2+t^2}) \right]. \end{aligned} \tag{20}$$

According to the Riemann-Lebesgue lemma, the asymptotic expansion for $A_{\rm reg}(0,\eta)$ as $\eta \to \infty$ has the form

$$\frac{A_{\rm reg}(0,\eta)}{\mu_0 I_0} \sim -\frac{1}{\pi^2\eta^2} \int_0^\infty dt q^{(1)}(0,t) + \frac{1}{\pi^2\eta^4} \int_0^\infty dt q^{(3)}(0,t) - \frac{1}{\pi^2\eta^6} \int_0^\infty dt q^{(5)}(0,t), \tag{21}$$

where

$$\begin{aligned} q^{(1)}(0,t) &= -\,{\rm csch}(\alpha) K_0(\sqrt{\alpha^2+t^2}) - \alpha \coth(\alpha)\,{\rm csch}(\alpha) \frac{K_1(\sqrt{\alpha^2+t^2})}{\sqrt{\alpha^2+t^2}}, \\ q^{(3)}(0,t) &= 3\,{\rm csch}(\alpha)[1 + t^2 + \alpha\coth(\alpha) - 2\coth^2(\alpha)] K_0(\sqrt{\alpha^2+t^2}) \\ &+ \frac{3}{4}\alpha\,{\rm csch}^4(\alpha) \frac{K_1(\sqrt{\alpha^2+t^2})}{\sqrt{\alpha^2+t^2}} \\ &\times[-(8+t^2)\cosh(\alpha) + t^2\cosh(3\alpha) + 5\alpha\sinh(\alpha) + \alpha\sinh(3\alpha)], \\ q^{(5)}(0,t) &= -\frac{5}{8}\,{\rm csch}^5(\alpha) K_0(\sqrt{\alpha^2+t^2}) \\ &\times[4(15 - 6t^2 - t^4 + 3\alpha^2)\cosh(2\alpha) - (3 + 6t^2 - t^4 - 3\alpha^2)\cosh(4\alpha) \\ &\quad - 6(23 + 2t^2)\alpha\sinh(2\alpha) + 3(-1 + 2t^2)\alpha\sinh(4\alpha) \\ &\quad + 3(45 + 10t^2 + t^4 - 5\alpha^2)] \\ &- \frac{5}{16}\alpha\,{\rm csch}^6(\alpha) \frac{K_1(\sqrt{\alpha^2+t^2})}{\sqrt{\alpha^2+t^2}} \\ &\times[2(192 + 24t^2 + t^4 - 33\alpha^2)\cosh(\alpha) - 3(16t^2 + t^4 - 21\alpha^2)\cosh(3\alpha) \\ &\quad + (t^4 + 3\alpha^2)\cosh(5\alpha) - (378 + 84t^2)\alpha\sinh(\alpha) \\ &\quad - (135 - 18t^2)\alpha\sinh(3\alpha) + 3(1 + 2t^2)\sinh(5\alpha)]. \end{aligned}$$

Taking the integrals over t in Eq. (21) numerically, we obtain for $\alpha = 1$ the result quoted in Eq. (16).

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