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NUMERICAL RENORMGROUP ANALYSIS OF SELF-ORGANIZED CRITICALITY IN LOW TEMPERATURE CREEP MODEL[1]

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Success of renormalization group (RG) approach frequently depends on a proper functional form (probe function) used for renormalization. The behavior of coarse grained variables and their distributions under RG transformation have been numerically investigated for a stationary but non-equilibrium process of low-temperature creep. The simulation allows the investigation of a respective model at two levels (microscopic and coarse grained ones) simultaneously. A remarkable result of the model is that one-particle distributions for coarse (block) variables become Gaussians for block size 4 and larger. The investigation of dispersion dependence on block size has revealed a long-range correlation described by a critical index close to 5/3 (instead of 2 for independent variables). It is concluded that a probe function for many-body (many-particle) distribution at RG approach should be of Gaussian form.

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1 Introduction

Self-organized criticality (SOC) [1–7] and critical phenomena have common features showing long-range (infinite) correlation in space and time (1/f noise). Due to this similarity the method of renormalization group can be applicable to SOC. However this fraught with a fundamental difficulty. Critical phenomena are thermodynamically equilibrium states and could be described probabilistically by the Gibbs distribution based on the Hamiltonian of the system. By definition SOC [1,2] is a stationary but non-equilibrium state and its corresponding distribution function is not a priori known. So there is no object to which the procedure of renormalization group could be applied. The aim of the investigation is first to numerically establish the stability of characteristics of SOC evolution under RG transformation. The characteristics chosen are one-dimensional distribution densities. In relation to statistical description of many particle systems they are called below one-particle distributions.

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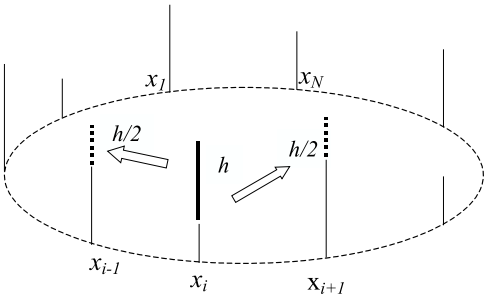

Fig. 1. N segments on the ring upon activation of the maximum height and conservation of the total height is a toy-model of a $1D$ low temperature creep. The activation comprises a redistribution of activated segment heights, decrease of the maximum height by a random value h, and transfer of its half to each neighbor.

2 Model

Low-temperature creep phenomena [5] were chosen for the numerical investigation. Evolution of the system comprises the transfer from a mechanically equilibrium state to another stable state overcoming a barrier due to some external reason (driving force and temperature). The low temperature limit means that the system moves over the weakest barrier in the variety of possible transitions. Dislocation movement in crystals, vortices in superconductors, domain walls in magnetics, grain boundaries in polycrystalls etc. could be physical prototypes of the model. It was shown earlier [5] that a toy-model could be considered instead of a "real" model. The toy-model is formulated as follows:

Consider N segments with heights x_i on a ring (Fig. 1). At the first step, a site with the maximum height should be found. Then a random piece h is cut from the maximum length. The piece is divided into two equal parts and the parts are added to neighbor sites. These steps are repeated many times, first to bring the system into a stationary state and then to calculate histograms and averages numerically. It is seen that the total height of all segments is constant upon evolution. The total height could be arbitrary and is supposed to be zero in the simulation.

For further interpretation it should be noted that the height has the meaning of force and the conservation law is a consequence of mechanical equilibrium when local forces (heights in the toy-model) compensate foe the external pressure multiplied by the system size.

3 Simulation

In this investigation, two kinds of histograms were calculated: the distribution of activated force (height) and distribution of the height (force) taken at random. Simultaneously the system was divided into blocks of size a (Fig. 2) and coarse-grained variables x_k^a were introduced

$$x_k^a = \frac{1}{a}\sum_i x_i \,.$$

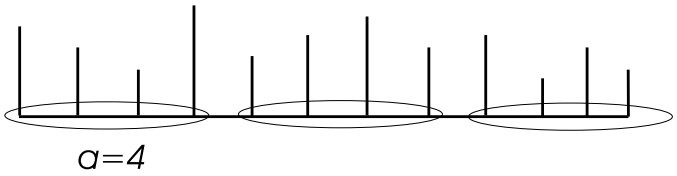

Fig. 2. Coarse-grained description is based on block values which are simply a mean value of the microscopic heights within a block of size a.

Index k denotes number of the block ($k = 1, 2, ...N/a$) and summation is performed within the k-th block (from $i = (k-1)a + 1$ till $i = ka$).

The simulation was performed at two levels, and the system evolved according to the microscopic rules described. But histogram accumulation was performed at the block level. Block size $a = 1$ corresponds to the microscopic level. At the microscopic level, an activated block is simply a segment with a maximum height. At the coarse grained level, an activated block can be not maximum. A block height does not change due to the conservation rule and, unlike microscopic level, a microscopic activation may not result in changes (decreasing) of the activated block height if microscopic activation occurs inside the block.

Histograms (distributions of probability density) of microscopic values (force taken at random and activated force) are shown in Fig. 3. The histograms display a picture common for extremal models picture [4,7] when two histograms are well separated at some critical value of force.

Fig. 4 shows histograms (distributions of probability density) of activated coarse-grained force for different block size values. The distributions become more narrow as the block size increase, the mean value of the activated force becomes simultaneously smaller and smaller. The distributions are presented in normalized form when random force is scaled with a standard deviation and the mean value is subtracted. The solid line is a standard Gaussian curve with the mean value equal to zero and standard deviation equal to unity. The block size dependence of dispersions (for activated block force and current block force) and the mean value of activated block force are shown in Fig. 5.

It is seen that coarse-grained variables are described by distribution very close to the Gaussian, beginning from a rather small block size, 4. Distributions of current block variable (force averaged over block taken at random) are also of Gaussian form with the mean value fluctuating near zero. Fig. 5 shows that dispersions of current and activated forces are equal. It is also seen that the dispersions decrease faster than the mean value. This means that the two distributions are well separated and the relative separation (ratio of mean value to standard deviation) increases with block size. The dashed and solid lines (in Fig.5) represent the approximating power laws with exponents equal to 1 (for mean value, dashed) and $f = 5/3$ (for dispersions, solid). Note that the block value is a sum of random values and, from Central Limiting Theorem, an exponent can be expected to be equal to 1 for dispersions. Non-classical behavior of the dispersions means that the forces inside a block are correlated and the dependence on block size contains information about the force to force correlation. Inverse proportionality of the activated mean value simply means that a block contains a number of forces which exceed the critical level and

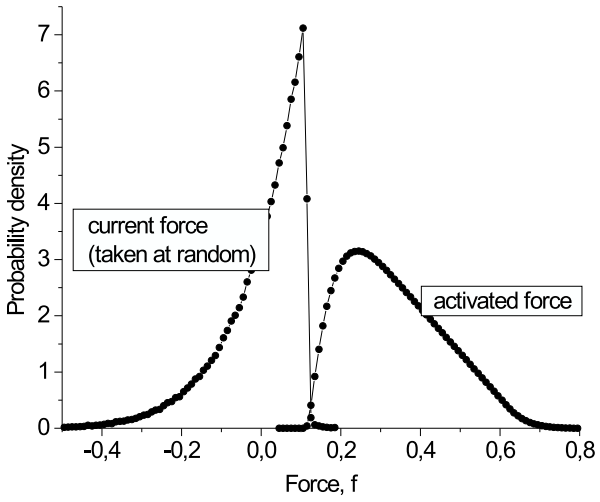

Fig. 3. Histograms of current force (taken at random along the ring) and maximum (activated) force. Separation of the histograms is a usual feature of extremal models like invasion percolation [3] and evolution model (Bak-Sneppen) [7] etc.

this number is not dependent on block size. The conclusion is consistent with the idea that in extremal SOC models [8] the number of active sites is finite and this number does not increase with size of the system.

4 Discussion

The results of numerical investigation of RG transformation in the model of low-temperature creep show that at least one-particle distributions demonstrate scaling behavior and the corresponding critical exponent is related to the long-range force to force spatial correlation.

However, a much more general conclusion about many-particle distribution could be made. Let us consider Gaussian distribution of N variables

$$p(x) = \sqrt{\frac{\beta}{(2\pi)^N}}\exp(-\frac{1}{2}\beta_{ik}x_i x_k);\ \ \beta = \det(\beta_{ik})\,.$$

Due to spatial uniformity of the system, the matrix β_{ik} should be a special matrix of a form in which elements in different rows are cyclically shifted

$$\beta_{ik}^{-1} = \left\| \begin{array}{ccccc} q_1 & q_2 & q_3 & \cdot & q_N \\ q_N & q_1 & q_2 & \cdot & q_{N-1} \\ q_{N-1} & q_N & q_1 & \cdot & q_{N-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ q_{N-3} & q_{N-2} & q_{N-1} & \cdot & q_1 \end{array} \right\| .$$

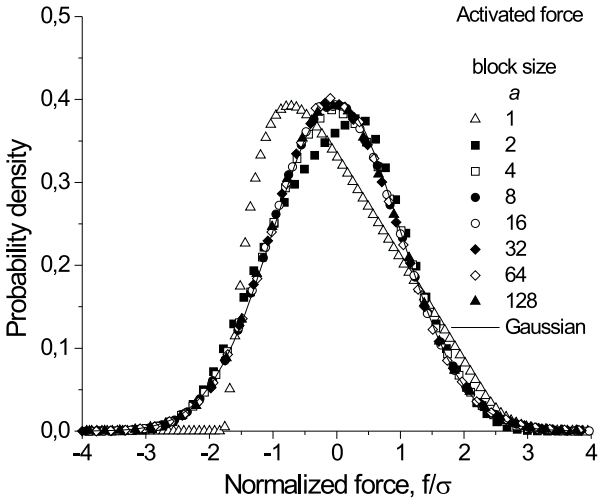

Fig. 4. Normalized histograms of activated coarse-grained force (points) for different block size ($a = 1, 2, 4, 8, 16, 32, 64, 128$). The solid curve is a Gaussian distribution with the zero mean and dispersion equal to unity.

The elements are related to the correlation coefficients $\langle x_i x_k \rangle = \beta_{ik}^{-1}$.

To obtain the distribution p_a on block variable x_a, $p(x)$ should be integrated over all N variables under the condition $x_a = \frac{1}{a}\sum_1^a x_i$.

The integration gives

$$p(x_a) = \frac{1}{\sqrt{2\pi}C_a} \exp[-\frac{1}{2}\frac{x_a^2}{C_a^2}] \,.$$

Dispersion C_a^2 is related to the original distribution and correlation coefficients

$$C_a^2 = \frac{1}{a}\sum_1^a \beta_{1k}^{-1} = \frac{1}{a}\sum_1^a q_i \,.$$

The relation can be reversed

$$q_k = (k+1)C_{k+1}^2 - kC_k^2 \,.$$

This results in an asymptotic relation for correlation coefficients considering the power law is observed in numerical calculations (Fig.5)

$$q_k \cong \frac{k+1}{(k+1)^f} - \frac{k}{k^f} \cong \frac{1}{k^f} \,.$$

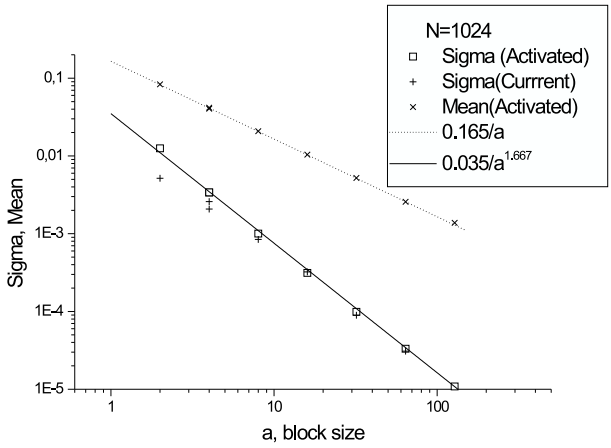

Fig. 5. Dependence of the mean value and dispersions on block size a.

5 Conclusion

Numerical calculations of one-particle distributions exhibit the stability of distributions under RG transformation and allow the conclusion that many-particle distribution of variables in models of low temperature creep which is Gaussian with long-range force to force correlations. The correlations are characterized by the critical exponent f with a numerical value close to $5/3$.

The general result concerning the functional form of many-particle distribution is essential for analytical approaches: the Gaussian function of many variables can now be used as a probe function in standard RG procedure when stability the functional form under RG transformation allows to determine values of critical exponents. In case of Gaussian probe function this means determination of the exponent related to force to force correlations.

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