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# ENERGY IN REBOUCAS-TIOMNO-KOROTKII-OBUKHOV AND GÖDEL-TYPE SPACE-TIMES IN BERGMANN-THOMSON'S FORMULATIONS

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We calculate the total energy (the matter plus fields) of the universe considering Bergmann-Thomson's energy-momentum formulation in both Einstein's theory of general relativity and tele-parallel gravity on two different space-times; namely Reboucas-Tiomno-Korotkii-Obukhov and the Gödel-type metrics. We also compute some kinematical quantities for these space-times and find that these space-times have shear-free expansion and non-vanishing four-acceleration and vorticity. Different approximations of the Bergmann-Thomson energy-momentum formulation in these different gravitation theories give the same energy density and agree with each other. The results advocate the importance of energy-momentum definitions.

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## 1 Introduction

The problem of energy localization is one of the oldest and most controversial problems which remain unsolved since the advent of Einstein's theory of general relativity [1]. Recently, this problem argued in tele-parallel gravity; It has been worked out by many physicists [2–10]. After Einstein's original work [11] on energy-momentum formulations, various definitions for energy-momentum densities were proposed: e.g. Tolman, Papapetrou, Landau-Lifshitz, Bergmann-Thomson, Møller, Weinberg, Qadir-Sharif and also tele-parallel gravity analogs of some of them. Except for the Møller formulation, these energy-momentum definitions are restricted to calculate the energy-momentum distribution in quasi-Cartesian coordinates. Møller proposed an expression which could be applied to any coordinate system. The notion of energy-momentum complexes was severely criticized for a number of reasons. First, the nature of a symmetric and locally conserved object is a non-tensorial one; thus its physical interpretation appeared obscure [12]. Second, different energy-momentum complexes could yield different energy-momentum distributions for the same gravitational backgrounds [13]. Finally, energy-momentum complexes

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were local objects while it was usually believed that the suitable energy-momentum of the gravitational field was only total, i.e. it cannot be localized [14]. For a long time, attempts to deal with this problem were made only by proposers of quasi-local approach [15, 16].

There have been several attempts to calculate energy-momentum densities by using these energy-momentum definitions associated with many different space-times [17–21]. In Ref. [17] Virbhadra showed that different energy-momentum formulations gave the same energy distribution as in the Penrose energy-momentum formulation by using the energy and momentum definitions of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class. Cooperstock and Israelit [22] found the zero value of energy for any homogenous isotropic universe described by the Friedmann-Robertson-Walker metric in the context of general relativity. This interesting result influenced some general relativists such as Rosen [23], Johri *et al.* [24], Banerjee and Sen [25]. Johri *et al.* found, using the Landau-Liftshitz energy-momentum definition, that the total energy of an Friedmann-Robertson-Walker spatially closed universe was zero at all times. Banerjee and Sen who investigated the problem of total energy of the Bianchi-I type space-times using the Einstein complex, obtained that the total energy was zero. This result agrees with the studies of Johri *et al.* since the line element of the Bianchi-I type space-time reduces to the spatially flat Friedmann-Robertson-Walker line element in a special case. Vargas [9] found, using the definitions of Einstein and Landau-Lifshitz in tele-parallel gravity, that the total energy was zero in Friedmann-Robertson-Walker space-times. This result agrees with the previous works of Cooperstock-Israelit, Rosen, Banerjee-Sen, Johri *et al*. Later on, Saltı and his collaborators considered different space-times for various definitions in tele-parallel gravity to obtain the energy-momentum distributions in a given model. First, Saltı and Havare [26] considered the Bergmann-Thomson's definition in both general relativity and tele-parallel gravity for the viscous Kasner-type metric. In another work, Saltı [27], using the Einstein and Landau-Lifshitz complexes in tele-parallel gravity for the same metric, found that total energy and momentum distributions were zero. Their results agree with previous results obtained in Refs. [9, 22–25]. At last, Aydogdu and Saltı [28] used the tele-parallel gravity analog Møller's definition for the Bianchi-I type metric and found that the total energy was zero.

The basic purpose of this paper is to obtain the total energy in Reboucas-Tiomno-Korotkii-Obukhov (RTKO) and the Gödel-type metrics by using the energy-momentum expression of Bergmann-Thomson in both general relativity and tele-parallel gravity. We will proceed according to the following scheme. In section 2, we give the RTKO and Gödel-type space-times and find some kinematical quantities associated with these metrics. In section 3, we give short brief of energy and momentum pseudo-tensors. Section 4 gives us the energy and momentum definition of Bergmann-Thomson in general relativity and its tele-parallel gravity analog, respectively. In section 5, we calculate the total energy densities. Finally, we summarize and discuss our results. Throughout this paper, the Latin indices ($i$, $j$, ...) represent the vector number, and the Greek ones ($\mu$, $\nu$,...) represent the vector components; all indices run from 0 to 3. We use units where $G = 1$ and $c = 1$.

## 2 The RTKO and Gödel-Type Space-Times

In this section we introduce the RTKO and Gödel-type metrics and then using these space-times we make some required calculations and find some kinematical quantities in these models.

### 2.1 The RTKO Model

The RTKO space-time is defined by the line element [29]

$$ds^2 = a^2(t)\left[-(dt + me^x dy)^2 + dx^2 + e^{2x}dy^2 + dz^2\right], \tag{1}$$

where $x$, $y$, $z$ are (real) spatial coordinates. The parameter $m$ is a constant that can be restricted to be positive without the loss of generality and is called rotation parameter. The line element which is given above does not contain closed time-like curves if and only if $m$ belongs to the interval [0,1) because it is only then that the metric induced on the sections of constant time is positively definite [30, 31]. For the line element (1), $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} = -a^2\delta^0_\mu\delta^0_\nu + a^2\delta^1_\mu\delta^1_\nu + (1-m^2)a^2e^{2x}\delta^2_\mu\delta^2_\nu + a^2\delta^3_\mu\delta^3_\nu - ma^2e^x[\delta^0_\mu\delta^2_\nu + \delta^2_\mu\delta^0_\nu], \tag{2}$$

and its inverse $g^{\mu\nu}$

$$g^{\mu\nu} = \frac{(m^2-1)}{a^2}\delta^\mu_0\delta^\nu_0 + \frac{1}{a^2}\delta^\mu_1\delta^\nu_1 + \frac{e^{-2x}}{a^2}\delta^\mu_2\delta^\nu_2 + \frac{1}{a^2}\delta^\mu_3\delta^\nu_3 - \frac{me^{-x}}{a^2}[\delta^\mu_0\delta^\nu_2 + \delta^\mu_2\delta^\nu_0]. \tag{3}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu. \tag{4}$$

Using this relation, we obtain the tetrad components

$$h^i{}_\mu = a\delta^i_0\delta^0_\mu + a\delta^i_1\delta^1_\mu + ae^x\delta^i_2\delta^2_\mu + a\delta^i_3\delta^3_\mu + mae^x\delta^i_0\delta^2_\mu \tag{5}$$

and their inverses are

$$h_i{}^\mu = \frac{1}{a}\delta^0_i\delta^\mu_0 + \frac{1}{a}\delta^1_i\delta^\mu_1 + \frac{e^{-x}}{a}\delta^2_i\delta^\mu_2 + \frac{1}{a}\delta^3_i\delta^\mu_3 - \frac{m}{a}\delta^2_i\delta^\mu_0. \tag{6}$$

After the pioneering works of Gamow [32] and Gödel [33], the idea of global rotation of the universe has become a considerably important physical aspect in the calculations. For the line element which describes the RTKO universe, one can introduce the tetrad basis

$$\theta^0 = adt + mae^x dy, \qquad \theta^1 = adx, \qquad \theta^2 = ae^x dy, \qquad \theta^3 = adz. \tag{7}$$

With the co-moving tetrad formalism, the kinematical variables of this model can be expressed solely in terms of the structure coefficients of the tetrad basis defined as

$$d\theta^\alpha = \frac{1}{2}\Sigma^\alpha_{\beta\gamma}\theta^\beta \wedge \theta^\gamma. \tag{8}$$

By taking the exterior derivatives of the tetrad basis which are given above and using the kinematics formulas [34]:

| | |
|---|---|
| four-acceleration vector: | $a_\mu = \Sigma^0_{\mu 0}$, |
| vorticity tensor: | $\omega_{\mu\nu} = \frac{1}{2}\Sigma^0_{\mu\nu}$, |
| expansion tensor: | $\xi_{\mu\nu} = \frac{1}{2}(\Sigma_{\mu 0\nu} + \Sigma_{\nu 0\mu})$, |
| expansion scalar: | $\xi = \Sigma^1_{01} + \Sigma^2_{02} + c^3_{03}$, |
| vorticity vector: | $\omega^1 = \frac{1}{2}\Sigma^0_{23}, \qquad \omega^2 = \frac{1}{2}\Sigma^0_{31}, \qquad \omega^3 = \frac{1}{2}\Sigma^0_{12}$, |
| vorticity scalar: | $\omega = \frac{1}{4}[(\Sigma^0_{23})^2 + (\Sigma^0_{31})^2 + (\Sigma^0_{12})^2]^{1/2}$, |
| shear tensor: | $\sigma_{\mu\nu} = \xi_{\mu\nu} - \frac{1}{3}\xi\delta_{\mu\nu}$, |

we find for the line element (1)

$$a_2 = \frac{2m\dot{a}}{a^2}, \qquad \omega^3 = \frac{2m}{a}, \qquad \xi = \frac{6\dot{a}}{a^2}, \qquad \sigma_{\mu\nu} = 0. \tag{9}$$

We see that the model given in (1) has shear-free expansion and we also note that this model describes space-time which has non-vanishing four-acceleration and vorticity.

### 2.2 The Gödel-type Model

In 1949, Gödel found a solution of Einstein's field equations with cosmological constant for incoherent matter with rotation [33]. It is certainly the best known example of a cosmological model which makes it apparent that general relativity does not exclude the existence of closed time-like world lines, despite its Lorentzian character which leads to the local validity of the causality principle. Gödel's cosmological solution has a well-recognized importance which has, to a large extent, motivated the investigations on rotating cosmological Gödel-type space-times and on causal anomalies within the framework of general relativity [35, 36]. In cartesian coordinates $x^\alpha = (t, x, y, z)$, the Gödel-type metrics are given by [37]

$$ds^2 = (dt - \sqrt{\sigma} a e^{mx} dy)^2 - a^2(dx^2 + (k-\sigma)e^{2mx}dy^2 + dz^2), \tag{10}$$

where $m, \sigma, k$ are constant parameters, and $a(t)$ is the time-dependent cosmological scale factor. For the line element (10), $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} = \delta^0_\mu\delta^0_\nu - a^2\delta^1_\mu\delta^1_\nu - a^2 k e^{2mx}\delta^2_\mu\delta^2_\nu - a^2\delta^3_\mu\delta^3_\nu - \sqrt{\sigma} a e^x[\delta^0_\mu\delta^2_\nu + \delta^2_\mu\delta^0_\nu], \tag{11}$$

and its inverse $g^{\mu\nu}$

$$\begin{aligned} g^{\mu\nu} = -\frac{k}{k+\sigma}\delta^\mu_0\delta^\nu_0 - a^{-2}\delta^\mu_1\delta^\nu_1 - \frac{e^{-2x}}{a^2(k+\sigma)}\delta^\mu_2\delta^\nu_2 - a^{-2}\delta^\mu_3\delta^\nu_3 \\ - \frac{\sqrt{\sigma}}{a(k+\sigma)}e^{-mx}[\delta^\mu_0\delta^\nu_2 + \delta^\mu_2\delta^\nu_0]. \end{aligned} \tag{12}$$

Tetrad fields are given by

$$g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu. \tag{13}$$

From this relation, we obtain the tetrad components:

$$h^i{}_\mu = \delta_0^i \delta_\mu^0 + a\delta_1^i \delta_\mu^1 + \sqrt{k+\sigma}ae^{mx}\delta_2^i \delta_\mu^2 + a\delta_3^i \delta_\mu^3 - \sqrt{\sigma}ae^{mx}\delta_0^i \delta_\mu^2. \tag{14}$$

and their inverses are

$$h_i{}^\mu = \delta_i^0 \delta_0^\mu + \frac{1}{a}\delta_i^1 \delta_1^\mu + \frac{e^{-mx}}{a\sqrt{k+\sigma}}\delta_i^2 \delta_2^\mu + \frac{1}{a}\delta_i^3 \delta_3^\mu - \sqrt{\frac{\sigma}{\sigma+k}}\delta_i^2 \delta_0^\mu. \tag{15}$$

Now we introduce tetrad bases by

$$\theta^0 = dt - \sqrt{\sigma}ae^{mx}dy, \qquad \theta^1 = adx, \qquad \theta^2 = \sqrt{k-\sigma}ae^{mx}dy, \qquad \theta^3 = adz, \tag{16}$$

and we find for the line element (10)

$$a_2 = \frac{\dot{a}}{a}\sqrt{\frac{\sigma}{\sigma-k}}, \qquad \omega^3 = \frac{m}{2a}\sqrt{\frac{\sigma}{\sigma-k}}, \qquad \sigma_{\mu\nu} = 0, \qquad \xi_{\mu\nu} = \frac{\dot{a}}{a}\delta_{\mu\nu}. \tag{17}$$

Using these results, we can say that the model given in (10) has shear-free expansion. And we also note that this model describes space-time which has non-vanishing four-acceleration and vorticity.

## 3 Energy-Momentum Pseudo-Tensors in General Relativity

The conservation laws of energy and momentum for an isolated systems are expressed by a set of differential equations. Defining $T_\beta^\alpha$ as the symmetric energy and momentum tensor (due to matter plus fields) the conservation laws are given by definition which is given below.

$$T^\alpha_{\beta,\alpha} \equiv \frac{\partial T^\alpha_\beta}{\partial x^\alpha} = 0, \tag{18}$$

where

$$\rho = T^0_0, \qquad j^i = T^i_0, \qquad p_i = -T^0_i \tag{19}$$

are the energy density, the energy current density, the momentum density, respectively, and Greek indices run over from the space-time labels while Latin indices take the values over the spatial coordinates.

Crossing from special to general relativity one assumes a simplicity principle which is called principle of minimal gravitational coupling. As a result of this, the equation which defines conservation laws is now

$$T^\alpha_{\beta,\alpha} \equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}T^\alpha_\beta\right) - \Gamma^\xi_{\beta\nu}T^\nu_\xi = 0, \tag{20}$$

where $g$ is the determinant of the metric tensor $g_{\mu\nu}(x)$. The conservation equation may also be written as

$$\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}T^\alpha_\beta\right) = \Gamma^\xi_{\beta\nu}T^\nu_\xi. \tag{21}$$

Here $\zeta_\beta = \Gamma^\xi_{\beta\nu} T^\nu_\xi$ is a non-tensorial object. Defining $\beta = t$ means that the matter energy is not a conserved quantity for the physical system[2]. From a physical point of view, the absence of energy conservation can be understood as a possibility of transforming matter energy into gravitational energy and vice versa. So, this remains a problem and it is widely believed that in order to solve it one has to take into account the gravitational energy [2, 4, 6, 7].

By a well-known procedure, the non-tensorial object $\zeta_\beta$ can be written as

$$\zeta_\beta = -\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\vartheta^\alpha_\beta\right), \tag{22}$$

where $\vartheta^\alpha_\beta$ are functions of the metric tensor and its first order derivatives. Therefore, the energy-momentum tensor of matter $T^\alpha_\beta$ is replaced by

$$\Omega^\alpha_\beta = \sqrt{-g}(T^\alpha_\beta + \vartheta^\alpha_\beta), \tag{23}$$

which is called the energy-momentum complex, since it is a combination of the tensor $T^\alpha_\beta$ and a pseudo-tensor $\vartheta^\alpha_\beta$, which describes the energy and momentum of the gravitational field. The energy-momentum complex satisfies a conservation law in the ordinary sense, i.e.

$$\Omega^\alpha_{\beta,\alpha} = 0, \tag{24}$$

and it can be written as

$$\Omega^\alpha_\beta = \Xi^{\alpha\lambda}_{\beta,\lambda}, \tag{25}$$

where $\Xi^{\alpha\lambda}_\beta$ are the super-potentials and they are the functions of the metric tensor and its first derivatives.

It is obvious that the energy and/or momentum complexes are not uniquely determined by the condition in which usual divergence is zero since it can always add a quantity with an identically vanishing divergence to the energy-momentum complex.

## 4 Bergmann-Thomson's Energy and Momentum Formulation

In this section, we introduce Bergmann-Thomson energy-momentum formulation. First, we give this formulation in general relativity and then we give its tele-parallel gravity version.

### 4.1 Bergmann-Thomson's energy-momentum formulation in general relativity

The energy-momentum prescription of Bergmann-Thomson [5] is given by

$$\Lambda^{\mu\nu} = \frac{1}{16\pi}\Pi^{\mu\nu\alpha}_{\ \ \ \ ,\alpha}, \tag{26}$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta}V^{\nu\alpha}_\beta \tag{27}$$

[2] It is possible to restore the conservation law by introducing a local inertial system for which at a specific space-time point $\zeta_\beta = 0$, but this equality by no means holds in general.

with

$$V_{\beta}^{\nu\alpha} = -V_{\beta}^{\alpha\nu} = \frac{g_{\beta\xi}}{\sqrt{-g}} \left[-g \left(g^{\nu\xi} g^{\alpha\rho} - g^{\alpha\xi} g^{\nu\rho}\right)\right]_{,\rho} . \tag{28}$$

The Bergmann-Thomson energy-momentum prescription satisfies the local conservation laws

$$\frac{\partial \Lambda^{\mu\nu}}{\partial x^{\nu}} = 0 \tag{29}$$

in any coordinate system. The energy and momentum (energy current) density components are respectively represented by $\Lambda^{00}$ and $\Lambda^{a0}$. The energy and momentum components are given by

$$P^{\mu} = \int\int\int \Lambda^{\mu 0} dx dy dz. \tag{30}$$

For the time-independent metric one has

$$P^{\mu} = \frac{1}{16\pi} \int\int \Pi^{\mu 0 a} \kappa_a dS. \tag{31}$$

Here, $\kappa_{\beta}$ is the outward unit normal vector of the infinitesimal surface element $dS$; $P^i$'s are the momentum components $P^1$, $P^2$, $P^3$ and $P^0$ is the energy.

### 4.2 Bergmann-Thomson's energy-momentum formulation in tele-parallel gravity

Tele-parallel gravity is an alternative approach to gravitation [38] which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry [39]. In this theory, gravitation is attributed to torsion [40], which plays the role of force [41], whereas the curvature tensor vanishes identically. The fundamental field is represented by a nontrivial tetrad field, which gives rise to the metric as a by-product. The last translational gauge potentials appear as the nontrivial part of the tetrad field, thus induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting point of tele-parallel gravity is that, due to gauge structure, it can reveal a more appropriate approach to consider the same specific problem. This is the case, for example, of the energy-momentum problem, which becomes more transparent when considered from the tele-parallel point of view.

The energy-momentum complex of Bergmann-Thomson in tele-parallel gravity [9] is given by

$$hB^{\mu\nu} = \frac{1}{4\pi} \partial_{\lambda} (g^{\mu\beta} U_{\beta}{}^{\nu\lambda}), \tag{32}$$

where $h = \mathtt{det}(h^{a}{}_{\mu})$ and $U_{\beta}{}^{\nu\lambda}$ is the Freud's super-potential

$$U_{\beta}{}^{\nu\lambda} = h S_{\beta}{}^{\nu\lambda}. \tag{33}$$

Here, $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = k_1 T^{\mu\nu\lambda} + \frac{k_2}{2} (T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{k_3}{2} (g^{\mu\lambda} T^{\beta\nu}{}_{\beta} - g^{\nu\mu} T^{\beta\lambda}{}_{\beta}) \tag{34}$$

with $k_1$, $k_2$ and $k_3$ being the three dimensionless coupling constants of tele-parallel gravity [40]. For the tele-parallel equivalent of general relativity, the specific choice of these three constants is

$$k_1 = \frac{1}{4}, \qquad k_2 = \frac{1}{2}, \qquad k_3 = -1. \tag{35}$$

To calculate this tensor we must calculate Weitzenböck connection first

$$\Gamma^{\alpha}{}_{\mu\nu} = h_a{}^{\alpha}\partial_\nu h^a{}_{\mu}, \tag{36}$$

and after this we get torsion of the Weitzenböck connection

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda}. \tag{37}$$

For the Bergmann-Thomson complex, we have

$$P_\mu = \int_\Sigma hB^0{}_\mu dxdydz, \tag{38}$$

where $P_i$'s are the momentum components $P_1$, $P_2$, $P_3$, while $P_0$ gives the energy, and the integration hyper-surface $\Sigma$ is described by $x^0 = t =$ constant.

## 5 The total energy of the universe in the RTKO and Gödel-Type Metrics

This section gives us the total energy of the universe based on the RTKO and Gödel-type metrics in both general theory of relativity and tele-parallel gravity.

### 5.1 Solutions in the RTKO model

Considering the line element (1) for the equations (27) and (28), the required components of $\Pi^{\mu\nu\alpha}$ are

$$\Pi^{000} = 0, \qquad \Pi^{001} = 2(m^2-1)e^x. \tag{39}$$

Substituting these results into (26), we find that

$$\Lambda^{00} = \frac{(m^2-1)e^x}{8\pi}. \tag{40}$$

Using equations (5) and (6), we find the non-vanishing components of the Weitzenböck connection

$$\Gamma^0_{00} = \Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \frac{\dot{a}}{a}, \qquad \Gamma^2_{21} = 1, \tag{41}$$

where $\dot{a}$ indicates derivative with respect to $t$. The corresponding non-vanishing torsion components are then

$$T^1_{01} = T^2_{02} = T^3_{03} = \frac{\dot{a}}{a}, \qquad T^2_{12} = 1. \tag{42}$$

Taking these results into equation (34), the non-zero components of the tensor $S_\mu{}^{\nu\lambda}$ are

$$S^{001} = \frac{(m^2-1)}{2a^4}, \tag{43}$$

$$S^{002} = \frac{2m\dot{a}(m^2-1)e^{-x}}{2a^5}, \tag{44}$$

$$S^{012} = \frac{me^{-x}}{2a^4}, \tag{45}$$

$$S^{201} = \frac{-me^{-x}}{2a^4}, \tag{46}$$

$$S^{313} = \frac{-1}{2a^4}, \tag{47}$$

$$S^{101} = (1-m^2)\frac{\dot{a}}{a^5}, \tag{48}$$

$$S^{202} = (1-m^2)\frac{\dot{a}e^{-2x}}{a^5}, \tag{49}$$

$$S^{303} = (1-m^2)\frac{\dot{a}}{a^5}. \tag{50}$$

Now, using equation (32) with (33), the total energy and non-vanishing momentum components are

$$hB^{00} = \frac{(m^2-1)e^x}{8\pi}. \tag{51}$$

### 5.2 Solutions in the Gödel-type model

To obtain energy density, we can substitute the line element (1) into equations (27) and (28). The required components of $\Pi^{\mu\nu\alpha}$ are obtained as

$$\Pi^{000} = 0, \qquad \Pi^{001} = -\frac{2kmae^{mx}}{(k+\sigma)^{\frac{1}{2}}}. \tag{52}$$

Substituting these results into (26), we get

$$\Lambda^{00} = -\frac{kmae^{mx}}{8\pi(k+\sigma)^{\frac{1}{2}}}. \tag{53}$$

Considering equations (5) and (6), the non-vanishing Weitzenböck connection components are

$$\Gamma^1{}_{10} = \Gamma^2{}_{20} = \Gamma^3{}_{30} = \frac{\dot{a}}{a}. \tag{54}$$

The corresponding non-vanishing components of torsion are found:

$$T^1{}_{01} = T^2{}_{02} = T^3{}_{03} = \frac{\dot{a}}{a}, \qquad T^2{}_{12} = m. \tag{55}$$

Using these results and equation (34), the non-vanishing components of the tensor $S_{\mu}{}^{\nu\lambda}$ are

$$S^{001} = -\frac{km}{2(k+\sigma)a^2}, \tag{56}$$

$$S^{002} = \frac{3k\dot{a}\sqrt{\sigma}}{2a^2(k+\sigma)^2}e^{-mx}, \tag{57}$$

$$S^{012} = -\frac{m\sqrt{\sigma}}{2(k+\sigma)a^3}e^{-mx}, \tag{58}$$

$$S^{313} = \frac{k_3}{2a^4}, \tag{59}$$

$$S^{101} = \frac{\dot{a}k}{a^3(k+\sigma)}, \tag{60}$$

$$S^{202} = \frac{ke^{-2mx}}{a^3(k+\sigma)^2}, \tag{61}$$

and

$$S^{303} = S^{101}, \qquad S^{201} = S^{012}, \tag{62}$$

and, if we use (32) and (33), the total energy and non-vanishing momentum components are

$$hB^{00} = -\frac{kmae^{mx}}{8\pi(k+\sigma)^{\frac{1}{2}}}. \tag{63}$$

## 6 Summary and Conclusions

The definition of energy-momentum localization in the general theory of relativity has been very exciting and interesting; however, it has been associated with some debate. Recently, some researchers have been interested in studying the energy content of the universe in various models.

Objective of the present paper is to show that it is possible to solve the problem of the energy localization in both general theory of relativity and tele-parallel gravity by using the energy-momentum formulations. First, we found some kinematical quantities of the universe based on the RTKO and Gödel-type line elements and after these calculations of the total energy (due to matter plus fields), we considered two different approaches of the Bergmann-Thomson energy-momentum definition. Finally, we found that: *(a)* the RTKO and Gödel-type models describe the space-times which have shear-free expansion, non-vanishing four-acceleration and vorticity, *(b)* the total energy distribution in Bergmann-Thomson's formulation is found exactly the same in both of these different gravitation theories and they agree with each other, *(c)* the results advocate the importance of energy momentum complexes.

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