HADRONG PRODUCTION OF η–MESONS: RECENT RESULTS AND OPEN QUESTIONS

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Recent insights and open questions connected with the production of η–mesons from hadrons are presented. The discussion includes the ηN, ηNN, ηd and η³He systems with emphasis on final state interactions.

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1 Introductory remarks

In recent years a large number of high accuracy data was published on the production of η–mesons in various reactions, such as πN → ηN [1, 2], γN → ηN [3], NN → NNη [4, 5], pn → dη [6], γd → NNη [7], γ³He → η³He [8], pd → η³He [9], as well as data on heavier nuclei that I will not discuss here.

Unfortunately, at present theory is far from being equally accurate and thus we are now pushed to identify the class of questions that can be currently addressed theoretically in a controlled way and to investigate those. In this article I will present my personal point of view of what should be studied in the coming years in respect to η production from hadrons. The eta–network provides the ideal ground for this enterprise.

It is well known that the production of eta mesons from single nucleons is dominated by the resonance \( S_{11}(1535) \) irrespective of the probe. Thus investigating η production off hadrons means to some extent to study the \( S_{11} \) in various settings. Especially since the nature of this resonance is heavily debated in the literature (see also next section and references given there), a systematic study of η production in various environments is of high importance.

2 Production from single nucleons and the \( S_{11} \)

Unfortunately it is not yet possible to derive hadron spectra from QCD directly, although large progress has been made in lattice approaches recently (see, e.g., Refs. [10]). We therefore do not yet understand how nature produces hadrons out of quarks. Probably the most prominent example that highlights our degree of ignorance is the spectrum of the lightest baryons.

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The problem is reflected in the ordering of the various states: The non–relativistic quark model in its original formulation — based on a harmonic oscillator potential [11] — predicts an arrangement of states with alternating parities. However, in the baryon spectrum the first positive parity excitation of the nucleon (the so called Roper resonance) is lighter than the first negative parity excitation — the \( S_{11}(1535) \). This is illustrated in Fig. 1.

The inclusion of an instanton induced interaction in the quark–quark potential improves the picture, however, without changing the order of the lightest states [12]. To my understanding there are so far two possible explanations of the spectrum in the literature, one, where the Roper is generated dynamically and thus interpreting the \( P_{11}(1710) \) as the first quark state [13,14], and another, where the ordering of states was changed by introducing a flavour–dependent quark–quark interaction [15].

At present it is not clear what the connection between these two pictures is — if such a connection exists at all. Recently the whole situation became even more confusing for it was claimed that based on chiral dynamics the \( S_{11}(1535) \) appears as a dynamically generated state [16,17]. A look at the baryon spectrum — Fig. 1 — reveals that, if confirmed, this puts the quark model in a complicated situation for now the order of states is in even more severe disagreement with its predictions and it is hard to imagine that a flavour–dependent interaction can overcome the huge gap between the 1650 and the 1440. It is therefore of high importance to understand the nature of the low lying resonances for this promises deep insights into how Nature makes hadrons.

How is it possible to distinguish molecular states and compact quark states? Based on an old proposal by Weinberg [18] in Ref. [19], in line with a series of older works [20], we identified the energy dependence of the elastic scattering amplitude in the molecule forming channel as a key quantity — especially the effective range in that channel should be large (in units of the range of forces) and negative for a quark state and small and positive for a molecule. The method applies if the binding energy of the state is significantly smaller than any other scale of the problem. In particular the inelastic threshold needs to be far away. Thus, to understand if the \( S_{11} \) is an \( \eta N \) molecule, experimental information is needed on elastic \( \eta N \) scattering. Unfortunately this channel is not directly accessible. Various theoretical analyses of the non–diagonal channels...
lead to only very weak constraints of, e.g., the \( \eta N \) scattering length: a compilation presented in Ref. [21] gives

\[ a_{\eta N} = (0.2 - 1.1, 0.26 - 0.35) \text{ fm}, \]

where the first range refers to the real part and the second to the imaginary part. An imaginary part to a scattering length arises in the presence of inelastic channels. Here these are the \( \pi N \) and the \( \pi \pi N \) channels. An improved analysis using all available data is therefore urgently called for. Note that an updated analysis for \( \pi N \) scattering is currently under way [22].

Experimental information on the \( S_{11}(1535) \) can be derived from data on \( \pi N \rightarrow \pi N, \pi N \rightarrow \eta N \) and \( \pi N \rightarrow \pi \pi N \) as well as the corresponding \( \gamma \) induced reactions to control systematics. One may ask whether the two–pion channels are really necessary — after all the particle data booklet only lists a branching ratio of 1–10 % for all the two–pion channels. However, a direct comparison of the total cross section for \( \pi N \rightarrow \eta N \) and the inelasticity of \( \pi N \) scattering in the \( S_{11} \) channel reveals a need for these. Using the optical theorem, one can directly convert the inelasticity \( \eta \) to the inelastic cross section. Based on the values for momentum and inelasticity as given in Ref. [23] we find at \( E_{\text{tot}} = 1540 \text{ MeV} \) \( \sigma_{\text{inel}}^{\pi N} = 2\pi(1 - \eta^2)/3k_1^2 = 3.5 \text{ mb} \), where \( k_1 \) denotes the incoming pion momentum. If the \( \eta N \) channel would be the only inelastic channel that couples to \( \pi N \) in the \( S_{11} \)–partial wave, these 3.5 mb need to agree to the peak value of the \( \pi N \rightarrow \eta N \) cross section. However, the measured value is significantly lower. The situation is illustrated in Fig. 2, where the results of Ref. [25] are shown. Note, in this work the two pion channel was parametrized as \( \pi \Delta \), however, at present it is not possible to decide on what the prominent \( \pi \pi N \) channel is. Thus a proper inclusion of the two pion channels is necessary, as was stressed in Refs. [24–26].

To summarize the investigations based on reactions on a single baryon we state that in order better to understand the nature of the \( S_{11}(1535) \) both high accuracy data and theoretical studies are necessary to derive constraints on the \( \eta N \) scattering parameters.

In addition, it is expected that a (loosely bound) molecule and a compact quark state behave very different in the presence of additional baryons. This field is unfortunately still lacking a systematic approach at present, although microscopic calculations are available for both the \( \eta d \) [27] and the \( \eta^3 \text{He} \) [28] system. In what follows I will briefly present and discuss on a qualitative level some phenomena recently observed in these systems.

3 The reaction \( NN \rightarrow \eta NN \)

Unpolarized data are available for total and differential cross sections for \( pp \rightarrow ppp \eta \) [4] and for total cross sections for \( pp \rightarrow pn\eta \) [5] and \( pp \rightarrow dp \) [6]. For \( pp \rightarrow ppp \eta \) analyzing powers were measured as well [29].

The energy dependence of the total cross sections for the various eta production channels in \( NN \) collisions is shown in Fig. 3. The curves are the results of a microscopic calculation using two different models for the \( NN \) interaction [26]. This calculation includes the \( \eta N \) interaction only to leading order. The picture nicely illustrates that it is necessary to go beyond perturbation theory for the \( \eta NN \) system to understand the energy dependence at low energies. This was done in Ref. [30] for the channel \( pp \rightarrow ppp \eta \) and in Refs. [27, 31] for \( pp \rightarrow dp \eta \) and it was indeed possible to describe the energy dependence of the various channels, once few body equations were employed for the final state interaction.
However, it turned out that the $pp$ invariant mass spectrum measured for $pp \rightarrow ppp$ [4] can not be described in this way — the data are shown in Fig. 4. The calculation that was capable to describe the energy dependence of the total cross section quite well, failed to describe the invariant mass spectrum [30]. The model results for this observable were quite close to the dashed line in the figure that was calculated under the assumption that only the $NN$ FSI distorts the $NN$ $S$–waves. To resolve the discrepancy it was speculated that there might be a significant $\eta N$ interaction [32], not captured by the model of Ref. [30]. On the contrary, Deloff argued that a significant energy dependence of the production operator is possible [33], and in Ref. [34] a sizable contribution of $NN P$–waves was proposed as a possible solution for the puzzle. Note that even isotropic angular distributions can be compatible with $NN P$–waves. All that can be read off the $pp\eta$ invariant mass spectrum is that a term quadratic in the final momentum is needed to explain the data (c.f. Fig. 4). However, such a dependence would emerge from all three explanations. Fortunately it is possible to isolate the $NN P$–waves by a double polarized measurement. For this it is sufficient to measure $A_{xx}$ as well as the differential cross section $\sigma_0$, since

$$\frac{3}{2} \sigma_2 = \sigma (1 + A_{xx}) = 2 \sigma (\uparrow \downarrow),$$

where the arrows indicate that both the spin of the beam as well as the target are aligned perpendicular to the beam. Consequently the spin–singlet initial state — and therefore neither $^1S_0 \rightarrow ^3P_0 s$ nor $^1D_2 \rightarrow ^3P_2 s$ — does not contribute to $\frac{3}{2} \sigma_2$. Note the observable is the same as that which can be used to measure the parity of narrow resonances [36].

Footnote: Here we use the notation $^{2S+1}L_J \rightarrow ^{2S'+1}L'_J l_l$, where $S, L, J (S', L', J')$ denote spin, angular momentum, and total angular momentum of the initial (final) $NN$ pair; $l_l$ denotes the angular momentum of the outgoing $\eta$–meson with respect to the $NN$ system. For a review of the selection rules see Ref. [35].
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Fig. 3. Energy dependence of the total cross sections for the various $\eta$ production channels. The different curves are results employing different $NN$ wave functions in a microscopic calculation for the different channels. The picture is taken from Ref. [26].

It should be clear that only when the partial wave decomposition of the $pp\eta$ spectrum is known, can we understand the role the $\eta N$ interaction plays here. Especially the fact that the calculation of Ref. [30] failed to describe the spectrum indicates that a lot is still to be learned from the reactions $NN \rightarrow NN\pi$. For example, in Ref. [34] it is shown that the requirement to describe the $pp\eta$ spectrum by $NN P$–waves strongly constrains the $NN \rightarrow NN^*(1535)$ transition potential.

4 The reactions $pd \rightarrow ^3\!He$ and $\gamma^3\!He \rightarrow ^3\!He$

The most prominent effect of the $\eta$–few–nucleon interaction can be seen in $\eta$–nucleus systems, like $\eta d$ [6], $\eta^3\!He$ [9], and $\eta\alpha$ [37].

Phenomenological investigations [38–40] as well as microscopic calculations for $\eta$–nucleus interactions exist from various groups [28] — some of these will be presented in the contribution by T. Peña to these proceedings. Thus here I will more focus on those features of the systems $\eta d$ and $\eta^3\!He$ that can be directly read from the data. The tool we use is a final state interaction enhancement factor $f(q)$, where $q$ denotes the momentum of the final state particles in the center of mass. According to Watson and Midgal [41] in the scattering length approximation

$$f(q) = \frac{1}{1 - i a q} = \frac{1}{1 - ia_R q + a_I q},$$

where $a_R$ ($a_I$) denotes the real (imaginary) part of the $\eta$–nucleus scattering length. As in case of the elementary amplitude (c.f. Eq. (1)) $a$ is a complex–valued quantity because of the presence of inelastic channels. Note that one should be careful in interpreting the value of $a$ extracted
from a fit to production data as the scattering length. It has been known for a long time that there is a well defined connection between the parameters of elastic scattering and the effect of final state interactions. This connection is put on a solid theoretical basis using dispersion theory [42]. However, a direct proportionality between the energy dependence of scattering and production only holds if the scattering length is significantly larger than the next term in the effective range expansion — the effective range. In addition, a systematic study of final state interaction effects revealed that the Watson formula tends to give a scattering length that is too large; however, there is a clear correlation between the scattering length as derived from elastic scattering and that extracted from the production reaction [43]. One should also stress that in the presence of inelasticities it is not even possible to derive an expression for the final state interaction effect in closed form. Thus the results should be taken on a qualitative level.

To study the singularity structure of Eq. (2) we need to analytically continue the momentum into the complex plane. The physical sheet is given by those momenta with positive imaginary parts — the unphysical one by negative imaginary parts. The sign of the imaginary part of the scattering length, on the other hand, is fixed to be positive through unitarity. It is the sign of the real part that decides, whether the nearby pole we want to investigate refers to a bound state or a virtual state. Thus only a negative real part refers to a bound state\(^4\). Haider and Liu [38] pointed out that for the existence of a bound state the additional condition \(a_f < |a_R|\) is to hold.

How can we measure the sign of the real part? In a cross section measurement what typically

\(^4\)Unfortunately there are different sign conventions present in the literature. We here use that of Goldberger and Watson [42], which is common for meson–nucleon systems. Note, however, for nucleon–nucleon scattering the scattering length is traditionally defined with a relative minus sign compared to the convention used here.
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Fig. 5. Result for the fit of the FSI formula of Eq. (2) to the world data on the reaction $pd \to \eta^3\text{He}$. The figures are taken from Ref. [44], where also a complete list of experimental references is given. In the left panel the data for the amplitude are shown as well as our best fits. For comparison also data on $pd \to \pi^0\text{He}$ are given. The right panel shows the confidence levels for the extracted real and imaginary part of the scattering length.

enters observables close to the threshold is $|f(q)|^2$. Then, above threshold, we get

$$|f(q)|^2 = \frac{1}{1 + 2a_Iq + |a|^2q^2}, \quad \text{with } q = \sqrt{2\mu E},$$

where $E$ denotes the kinetic energy of the final system with respect to the $\eta$–nucleus threshold and $\mu$ is the corresponding reduced mass. Therefore above threshold $E > 0$. Thus any measurement above the $\eta$–nucleus threshold is sensitive only to the magnitude of the real part, but not to its sign; measurements above threshold are necessary to pin down the absolute values of the real and the imaginary part of the scattering length. It should be clear from the given formula that only measurements very close to the threshold allow one to disentangle $a_I$ from $a_R$.

On the other hand, below threshold ($E \leq 0$) we get

$$|f(q)|^2 = \frac{1}{1 + 2a_R\kappa + |a|^2\kappa^2}, \quad \text{with } \kappa = \sqrt{-2\mu E}.$$

Thus, now real and imaginary part have changed their roles. Since we know the sign of the imaginary part high accuracy measurements of the energy dependence of the $\eta$–nucleus amplitude above and below the eta threshold allows one to extract the sign of the real part of the scattering length [45].
But what does it mean to measure an amplitude below threshold? This can only work by identifying inelastic channels that show a significant coupling to the $\eta$–nucleus channel. We will discuss one example of this in detail below. However, we would first like briefly to comment on the expected magnitude of the imaginary part. We know that there is a significant coupling of $\pi N \rightarrow \eta N$ (c.f. Eq. (1)) near threshold. One might therefore expect an imaginary part of the scattering length of the order of at least one fermi. In contrast to this, in a recent analysis of the world data base for $\eta^3$He, values for the imaginary part of the $\eta^3$He scattering length were extracted that were compatible with zero [44]. The result of this fit is illustrated in Fig. 5. The left panel shows the scattering amplitude, defined through

$$|f(q)|^2 = (q/k)\sigma_{tot}/4\pi,$$

where $k$ denote the initial cms momentum. From this fit we extracted

$$a = |4.3 \pm 0.3| + i(0.25 \pm 0.25) \text{ fm}.$$ 

The numbers given are in line with a recent K–matrix analysis [40].

Can we understand such a small imaginary part of the scattering length? The answer is yes. Let us start the discussion for simplicity with the $d$ system. The central observation was that the Pauli principal for few–nucleon systems also needs to hold for intermediate states [46]. Technically this is to be realized by a consistent inclusion of self energy diagrams and rescatterings. To understand the role of the $\pi NN$ intermediate state in the regime of a prominent $\eta d$ $s$–wave interaction, we observe, that the $\eta N \rightarrow \pi N$ operator must be an isovector acting on the nucleons. On the other hand, a $\eta N s$–wave necessarily connects to a $\pi N s$–wave and therefore the operator is spin independent. If such an operator acts on the deuteron wave function it forces the $NN$ pair into an isospin 1 state, but leaves the spin in the spin triplet. A spin triplet $NN$ pair with isospin 1, however, has necessarily odd angular momentum and therefore, to conserve parity, the pion must also be in a $p$–wave. As a consequence the potentially most prominent inelastic channel is blocked. As the deuteron is a prominent building block also of $^3$He this argument at least to some extent should hold as well. This was worked out in more detail in Ref. [47].

In the process of preparing this talk I observed an amusing similarity that I wanted to share. When plotted in the same figure both the $\eta d$ interaction and the $\eta^3$He interaction show a very similar energy dependence. This is illustrated in Fig. 6. The curves show a best fit to the $\eta d$ data as well as the best fit to the $\eta^3$He data, as explained above. This similarity might point at the conjecture used above, namely that the $\eta^3$He dynamics is at least to a large extent given by the interaction of the $\eta$ with the deuteron substructure of the $^3$He.

In contrast to the imaginary part of the scattering length, the real part, as obtained from the fit to the $pd \rightarrow \eta^3$He data, is quite sizable. Under the assumption that this large value is a signal of a bound state, in Ref. [48] the relation between the scattering length and the position of the bound state pole was investigated and values quite close to the threshold were extracted. However, to make this analysis useful we need to know, if a bound state exists or not.

Therefore we now turn to $\eta$–nucleus measurements below the $\eta$ threshold. As mentioned here we have to look at different channels. Experiments for this were proposed quite some time ago [49], but only realized recently. At COSY there was a measurement of the ratio $pd \rightarrow \pi^+ t$ to $pd \rightarrow \pi^0$He [50] and it could be shown that this ratio is indeed sensitive to the sign of the real
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Fig. 6. Energy dependence of the $\eta d$ (squares) and $\eta^3$He (dots) interaction as derived from cross section data of the reactions $pn \rightarrow \eta d$ [6] and $pd \rightarrow \eta^3$He [9]. The dashed line corresponds the best fit of the final state interaction formula Eq. (2) solely to the $\eta d$ data, whereas the solid curve corresponds to the best fit to the $\eta^3$He data.

part of the $\eta$–nucleus scattering length [45]. Since the existing data are not of sufficient accuracy and there is currently no new measurement planned, we will not describe this here in detail.

The TAPS collaboration at MAMI recently ran a quite successful experiment that clearly showed a strong $\eta^3$He interaction right below threshold [8]. What was measured was the reaction $\gamma^3$He $\rightarrow \pi^0 pX$ where, in order to find the signal, a cut was introduced to select only $\pi^0 p$ pairs that go out back–to–back in the centre–of–mass system. This cut was motivated by the observation that if there were a bound $\eta$–nucleus system the $S_{11}(1535)$ would certainly play a prominent role in it. As the measurement was performed right at threshold this $S_{11}$ should be at rest and correspondingly the outgoing $\pi^0 p$ pairs from its decay should go out back–to–back in the cms of the whole system. And indeed a clear peak structure could be identified in the data. The structure extracted, after subtraction of the neighbouring angular bins, is shown in the left panel of Fig. 7 as a function of the reduced photon energy. The vertical line indicates the position of the $\eta^3$He threshold. To analyze these data we may use the same formula for the final state interaction effects as used for other production reactions — Eq. (2) — with both imaginary part and real part of the scattering length as extracted from $pd \rightarrow \eta^3$He (c.f. Eq. (4)). This is possible because final state interactions are universal in large momentum transfer reactions.

There is one additional comment necessary before we can apply Eq. (2) to the TAPS data: there is in principle some interference with the background possible. Thus, what was identified as the resonance signal might well have some contribution from an interference term, and the full signal may be written as $N \left(2 \text{Re}(B f^{res}) + |f^{res}|^2\right)$, where $B$ is some complex number parameterizing that part of the background that is allowed to interfere with the resonance signal and $N$ is a measure of the total strength of the signal. Therefore, three different fits were performed: fit 1 included only the pure resonance signal ($B = 0$; only $N$ as a free parameter); fit 2 included only the interference term ($B \rightarrow \infty$; $N$ and the phase of $B$ as a free parameter); and fit 3 considered the full structure (thus here we have 3 free parameters: $N$, $|B|$ and the
Fig. 7. Comparison of the various fits to the data, as a function of the reduced photon energy $W$ defined in Ref. [8]. The left panel: binned as the data; the right panel: no binning. The solid (dashed) line corresponds to $a = (+4, 1)$ (or $a = (-4, 1)$) fm, and the dotted one to $a = (0, 3.5)$ fm. The vertical line indicates the position of the $^3$He threshold.

As it turned out, the $\chi^2$ per degree of freedom for the two scenarios (positive and negative real part of the scattering length) was almost the same in all three cases and thus for illustration in Fig. 7 we only show the results of the second fit, where the curves in the left panel correspond to the results after binning in accordance with that of the experiment and the right panel corresponds to the unbinned results. To keep the number of free parameters low we choose $a = (+4, 1)$ fm. In both figures the dashed line corresponds to a negative real part (indicating the existence of a bound state) and the solid line corresponds to a positive real part (indicating a virtual state). For comparison also a curve is shown that has the maximum imaginary part together with a vanishing real part, which is still compatible with a subset of the available $pd \rightarrow \eta^3$He data [44]. The fit gave a $\chi^2$ per degree of freedom of 1 for the latter case, whereas it was worse than 3 in the former. Thus the data prefer the solution that corresponds to a virtual state, although the existence of a bound state can not be excluded, given the quality of the data. Note, already in Ref. [48] the interpretation of the TAPS data as a bound state was questioned. Fortunately a new experiment will be performed soon. The expected much higher statistics promise in the near future an unambiguous decision on what scenario is realized: a bound state or a virtual state.

5 Summary

In this talk various aspects of $\eta$–meson production were discussed. The main issue is to investigate the $\eta$–nucleon interaction and in particular the $S_{11}$ resonance in various environments. These studies promise insights into not only the nature of this state but also into the existence of meson–nucleus bound states. Especially on the theory side a lot still needs to be done, but this field of research promises deep insight in strong interaction physics in the non–perturbative regime.
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